DTU Library

Risk Averse Bidding of Wind Power - Formulation and Properties

Jónsson, Tryggvi; Ryan, Sarah M.; Zugno, Marco; Pinson, Pierre

Publication date:
2012

Document Version
Publisher's PDF, also known as Version of record

Citation (APA):
Jónsson, T., Ryan, S. M., Zugno, M., & Pinson, P. (2012). *Risk Averse Bidding of Wind Power - Formulation and Properties*. Technical University of Denmark. D T U Compute. Technical Report, No. 2012-03

Risk Averse Bidding of Wind Power - Formulation and Properties

Tryggvi Jónsson, Sarah M. Ryan, Macro Zugno, Pierre Pinson

March 2, 2012

IMM-Technical Report-2012-03

Nomenclature

Main symbols:

ρ_k	Wind power producer revenues at trading period k
W_k	Wind power production at trading period k
π_k	Market price at trading period k
C_k	Negative wind power producer revenues due to imbalance at trading period k
ψ_k	Unit regulation costs for positive and negative imbalances at trading period k
$W^{(max)}$	Installed wind power capacity
r_k	Quantile of wind power distribution at trading period k
P_k	Probability of imbalance direction at trading period k
a_v	Parameter determining the width of the bound to the optimal bid in the decision space
a_p	Parameter determining the width of the bound to the optimal bid in the probability space

Superscripts:

$^{(S)}$	Referring to the day-ahead market
$^{(\uparrow/\downarrow)}$	Referring to the real-time market
$^{(\uparrow)}$	Referring to up-regulation in the real-time market
$^{(\downarrow)}$	Referring to down-regulation in the real-time market
*	Optimal
$\sim$	Contracted at the day-ahead market
$\frown$	Forecast

1 Introduction

The recent year's tremendous growth in installed wind power generation capacity worldwide has fuelled growing interest in how electricity generated by these plants is best offered to the market. A pioneer work on the topic is presented in Bremnes (2004) which shows analytically that the optimal bid for a wind energy producer in the day-ahead market is a particular quantile of a predictive generation density. Building further on these findings, the authors of this report present in Zugno et al. (2012) a case study where trading of wind power, entirely based on forecasts, during a 10 month period is simulated.

The biggest limitation of the strategy presented in Zugno et al. (2012) is that it only yields an optimal bid as long as the quantity traded is small enough not to affect the market outcome in any way. As soon as the strategy is adopted for a price making quantity, the deviations from the point forecast can act to increase the probability of deviations in that direction being penalised. Thus the effects of the strategy risk becoming exactly the opposite of what is intended. This situation can be the result of either a single price making producer adopting the strategy or prompted a collective adaptation of the strategy by a number of wind power producers (given that their production at any given time is highly correlated). Even though both of these scenarios have the same effect, the appropriate countermeasures are most likely different. Whereas a single price making producer has to explicitly consider the full extent of the feedback his actions create on the market, a small producer can still take advantage of the fact that the market is indifferent to his individual actions. His main concerns are therefore to add the knowledge that other small producers are also strategic in their bidding, e.g. by taking a more game theoretical approach to his bidding.

The strategy presented in Zugno et al. (2012) also pays no explicit regard to the risk involved. The hours of severe losses are merely avoided by restricting the deviation from the point forecast either in terms of percentage of the point forecast or in terms of probability. These constraints on the bid are imposed by the TSO although how much deviation would be accepted is not clear. The reasoning for regarding this choice as sufficient consideration for risk is twofold. Firstly, most likely will any popular restriction of the variance of the revenue would lead to larger deviations from the point forecast than allowed by the TSO. Secondly, the revenue risk is only relevant on a quarterly or annual basis and due to the many independent decisions made within one year (8760 on a non-leap year to be exact) that maximising expectations is sufficient.

Generally speaking however the need for some risk awareness is undisputed since applying the optimal quantile strategy directly without restrictions yields several hours of excessive losses during the test period in Zugno et al. (2012). Then due to the fact that the producers are only subject to downside risk these losses are not countered by huge gains. Moreover, even though the practical constraints seem to be sufficient in the case study there exist (in theory at least) scenarios where a producer might suffer large losses despite the aforementioned restrictions. This is because putting a limit on the deviation from the point forecast only addresses the part of the risk that arises from the uncertain production and not the part that is owed to the volatility of the regulation prices/penalties. In other words the possibility of a large loss arising from a small deviation times a large penalty is not addressed by the restrictions in Zugno et al. (2012). Thus excessive losses could be suffered e.g. in a more volatile market/period than the one considered in Zugno et al. (2012). This aside, the market feedback from a price maker's decisions might prompt an increased risk of excessive losses despite retaining the bid within a certain limit from the point forecast and market circumstances being similar to that in the previously mentioned case study. So in summary, the incorporation of a more comprehensive risk measure in the bidding strategy would strengthen the theoretical grounds for the strategy already developed and contribute to a more seamless adoption to other markets and extensions.

The aim of this report is to derive and analyse the methods for wind power trading which account for risk and are applicable for a price maker in the market. At first the general formulation of the risk averse bidding problem is derived analytically and the effect of the price taker assumption discussed. Similar to Morales et al. (2010) and Dent et al. (2011), the risk criteria considered is the Conditional Value-at-Risk (CVaR). Afterwards, an operational formulation of the problem in a stochastic programming framework is obtained and the generation of the scenarios necessary for solving it informally discussed. Finally the characteristics of such a strategy are demonstrated through a small case study and the resulting bids compared to the bids found by the strategy in Zugno et al. (2012). Readers of this report are assumed to have basic knowledge about the general functionalities of Nord Pool's markets and about strategic bidding of wind power. A more comprehensive introduction to these aspects is given in Zugno et al. (2012).

2 Being a Price Maker

Before going any further, spending a few words on what being a price maker or a price taker involves and why the concepts are relevant in the context of this report is in order. In a perfect competitive market, all participants are price takers since the do not affect the market outcome in any way by the actions. A price maker in a particular market on the other hand can influence the price on the market it is involved in by its trades. Since the potential influence on the market is owed (in this case) to a prominent share in the electricity supply the actual market power of a producer may vary both between the markets he's active on and might also be not constant in time. More precisely a wind power producer[1] with a production capacity that is large enough to affect the spot price in a single price area (say DK-1) may loose some of his impact during hours when there is no congestion on the transmission lines to the surrounding areas.

Furthermore, a producer's impact may change from being on the supply side in one market to being on the demand side in another. On the spot market though, it is clear that it is the volume bid to the market that affects the price as it is indeed that volume that enters the supply function and shifts it to the right. The same wind power producer and his actions on the spot market is a source for demand on the regulation market. That is their imbalances are what prompts the demand for regulation power. However at the same time this very same wind power affects the supply of regulating power indirectly as it has the ability to push less cost efficient yet flexible power generators to the right of the equilibrium price. So the impact on opposite sides of the supply/demand might act as counterbalancing to some extent. This is the case as long as the wind power producer can't be a direct supplier of regulating power. Finally it is important to realise that the market power of a particular collection of wind turbines is not the same on the spot market and the regulation market even in situations where the spot price is unique for the area in question. This is because the different parties are involved in the different markets (e.g. not everybody need to regulate).

So in conclusion, abandoning the price taker assumption prompts that the following relationships need to be accounted for:

- The impact of the bid quantity ($\widetilde{W}_k$) on the spot price ($\pi_k^{(S)}$) and the regulation price ($\pi_k^{(\uparrow/\downarrow)}$)/penalty ($\psi_k^{(\uparrow/\downarrow)}$)
- The impact of the price maker's imbalance ($W_k - \widetilde{W}_k$) on the regulation price/penalty through the demand for regulating power
- The impact of the price maker's imbalance on the regulation price/penalty through impact on the supply of regulating power

This aside, the regulation prices/penalties are known to be dependent on the spot prices. Now assuming that all the mentioned variables actually impact the prices the spot price should be written as a function of the bid wind power, $\pi_k^{(S)}(\widetilde{W}_k)$, and the regulation prices/penalties as a function of the bid wind, the produced wind power and the spot price $\pi_k^{(\uparrow/\downarrow)}(\widetilde{W}, W_k, \pi_k^{(S)})$/ $\pi_k^{(\uparrow/\downarrow)}(\widetilde{W}, W_k, \pi_k^{(S)})$. Obviously, it has to be investigated whether accounting for all these relationships is necessary but for the time being we will assume that to be the case.

3 Problem Formulation

In this report we continue along the lines of Zugno et al. (2012) and consider the bidding decision to be taken for each hour individually. This is in contrast to the formulations of Morales et al. (2010) and Dent et al. (2011) which consider all 24 bidding decisions for the

[1]or an aggregation of a number of producers with highly correlated production/behaviour

upcoming day simultaneously. A further discussion on the differences between those two approaches in theory and practise is given in the final section of this report. In this section analytical expressions for the expected revenue and the CVaR are derived to the extent possible. The formulation presented here is mainly adopted from Rockafellar and Uryasev (2000, 2002); Schultz and Tiedemann (2006); Chen et al. (2009).

As shown in Zugno et al. (2012) the revenue ρ_k for hour k can be written in the following ways:

$$\begin{aligned}
\rho_k &= \pi_k^{(S)}\widetilde{W}_k \\
&\quad + I\{W_k > \widetilde{W}_k\}(W_k - \widetilde{W}_k)\pi_k^{(\downarrow)} + I\{W_k < \widetilde{W}_k\}(W_k - \widetilde{W}_k)\pi_k^{(\uparrow)} \\
&= \pi_k^{(S)}W_k + (\widetilde{W}_k - W_k)\pi_k^{(S)} \\
&\quad + I\{W_k > \widetilde{W}_k\}(W_k - \widetilde{W}_k)\pi_k^{(\downarrow)} + I\{W_k < \widetilde{W}_k\}(W_k - \widetilde{W}_k)\pi_k^{(\uparrow)} \\
&= \pi_k^{(S)}W_k \\
&\quad - I\{W_k > \widetilde{W}_k\}(W_k - \widetilde{W}_k)\underbrace{(\pi_k^{(S)} - \pi_k^{(\downarrow)})}_{\psi_k^{(\downarrow)}} + I\{W_k < \widetilde{W}_k\}(W_k - \widetilde{W}_k)\underbrace{(\pi_k^{(\uparrow)} - \pi_k^{(S)})}_{\psi_k^{(\uparrow)}}.
\end{aligned} \tag{1}$$

Same place it is shown that a risk neutral producer is interested in the bid, $\widetilde{W}_k$ that maximises its expected revenues, i.e.

$$\widetilde{W}_k^{(*)} = \arg\max_{\widetilde{W}_k} \mathbb{E}[\rho_k|\widetilde{W}_k] \tag{2}$$

which is shown to be equivalent to solving

$$\widetilde{W}_t^{(*)} = \arg\max_{\widetilde{W}_t} \mathbb{E}\left[C_t^{(\uparrow/\downarrow)}\right] \tag{3}$$

where

$$C_t^{(\uparrow/\downarrow)} = I\{W_k < \widetilde{W}_k\}(W_k - \widetilde{W}_k)\psi^{(\uparrow)} - I\{W_k > \widetilde{W}_k\}(W_k - \widetilde{W}_k)\psi_k^{(\downarrow)} \tag{4}$$

if the market is unaffected by the producer's actions, i.e it is price taker.

Taking the expectation of Eq. (1) yields either

$$\begin{aligned}
\mathbb{E}[\rho_k|\widetilde{W}_k] &= \mathbb{E}[W_k\pi^{(S)}|\widetilde{W}_k] \\
&\quad - \mathbb{E}[I\{W_k > \widetilde{W}_k\}(W_k - \widetilde{W}_k)(\pi_k^{(S)} - \pi_k^{(\downarrow)})|\widetilde{W}_k] \\
&\quad + \mathbb{E}[I\{W_k < \widetilde{W}_k\}(W_k - \widetilde{W}_k)(\pi_k^{(\uparrow)} - \pi_k^{(S)})|\widetilde{W}_k] \\
&= \int_0^{W^{(max)}} W_k d\mathbb{P}_W \int_{\pi^{(S)}=-\infty}^{\infty} \pi_k^{(S)}(\widetilde{W})d\mathbb{P}_{\pi^{(S)}} \\
&\quad - \int_{\pi^{(S)}=-\infty}^{\infty} \int_{W_k=\widetilde{W}}^{W^{(max)}} \int_{\psi^{(\downarrow)}=0}^{\infty} [W_k - \widetilde{W}_k]\psi^{(\downarrow)}(\widetilde{W}_k, W_k, \pi_k^{(S)})d\mathbb{P}_{\psi^{(\downarrow)}}d\mathbb{P}_W d\mathbb{P}_{\pi^{(S)}} \\
&\quad + \int_{\pi^{(S)}=-\infty}^{\infty} \int_{W_k=0}^{\widetilde{W}} \int_{\psi^{(\uparrow)}=0}^{\infty} [W_k - \widetilde{W}_k]\psi^{(\uparrow)}(\widetilde{W}, W_k, \pi_k^{(S)})d\mathbb{P}_{\psi^{(\uparrow)}}d\mathbb{P}_W d\mathbb{P}_{\pi^{(S)}}
\end{aligned} \tag{5}$$

or

$$\begin{aligned}
\mathbb{E}[\rho_k|\widetilde{W}_k] &= \mathbb{E}[\widetilde{W}_k\pi_k^{(S)}|\widetilde{W}_k] \\
&\quad + \mathbb{E}[I\{W_k > \widetilde{W}_k\}(W_k - \widetilde{W}_k)\pi^{(\downarrow)}|\widetilde{W}_k] \\
&\quad + \mathbb{E}[I\{W_k < \widetilde{W}_k\}(W_k - \widetilde{W}_k)\pi^{(\uparrow)}|\widetilde{W}_k] \\
&= \widetilde{W}_k \int\limits_{\pi^{(S)}=-\infty}^{\infty} \pi_k^{(S)}(\widetilde{W})d\mathbb{P}_{\pi^{(S)}} \\
&\quad + \int\limits_{\pi^{(S)}=-\infty}^{\infty} \int\limits_{W_k=\widetilde{W}}^{W^{(max)}} \int\limits_{\pi^{(\downarrow)}=-\infty}^{\pi^{(S)}} [W_k - \widetilde{W}_k]\pi_k^{(\downarrow)}(\widetilde{W}_k, W_k, \pi_k^{(S)})d\mathbb{P}_{\pi^{(\downarrow)}}d\mathbb{P}_W d\mathbb{P}_{\pi^{(S)}} \\
&\quad + \int\limits_{\pi^{(S)}=-\infty}^{\infty} \int\limits_{W_k=0}^{\widetilde{W}} \int\limits_{\pi^{(\uparrow)}=\pi^{(S)}}^{\infty} [W_k - \widetilde{W}_k]\pi_k^{(\uparrow)}(\widetilde{W}, W_k, \pi_k^{(S)})d\mathbb{P}_{\pi^{(\uparrow)}}d\mathbb{P}_W d\mathbb{P}_{\pi^{(S)}}
\end{aligned} \tag{6}$$

where in both equations, $d\mathbb{P}_x = f(x)dx$ where in turn $f(x)$ is the probability density function (PDF) of x. In order to ease the notation in the following, $\pi_k^{(S)}(\widetilde{W})$ will be written as $\pi_k^{(S)}$. Likewise we will note $\pi_k^{(\uparrow/\downarrow)}(\widetilde{W}, W_k, \pi_k^{(S)})$ and $\psi_k^{(\uparrow/\downarrow)}(\widetilde{W}, W_k, \pi_k^{(S)})$ as $\pi_k^{(\uparrow/\downarrow)}$ and $\psi_k^{(\uparrow/\downarrow)}$ respectively.

In Eq. 5 the expectations in the first term can be separated without loss of generality. This is because the wind power actually produced has no impact on the prices. Instead, any correlation between W_k and $\pi_k^{(S)}$ can be explained by the correlation between $\widetilde{W}_k$ and $\pi_k^{(S)}$. The remaining terms are however inseparable in the general setting. Only if the producer is a price taker, the triple integrals can be separated into a wind power part and a price part as shown in Zugno et al. (2012). Between the formulations in Eqs. (5) and (6) the former has the appeal in the price taker case that the term for the spot market can be omitted in the optimisation making it a maximisation of the (non-positive) imbalance costs instead of the revenue. When the price taker assumption is abandoned however, the contribution from the spot market has to be considered as well. This in turn makes the choice between the two formulations less obvious.

A producer who is not completely risk neutral and characterises his risk in terms of CVaR seeks a solution to

$$\widetilde{W}_t^{(*)} = \arg\max_{\widetilde{W}_t} \left\{ \mathbb{E}\left[C_t^{(\uparrow/\downarrow)}\right] + \lambda \mathrm{CVaR}_\alpha \left[C_t^{(\uparrow/\downarrow)}\right] \right\} \tag{7}$$

or similarly in terms of the revenue as:

$$\widetilde{W}_t^{(*)} = \arg\max_{\widetilde{W}_t} \left\{ \mathbb{E}\left[\rho_t\right] + \lambda \mathrm{CVaR}_\alpha \left[\rho_t\right] \right\} \tag{8}$$

where in both cases $\mathrm{CVaR}_\alpha[\cdot]$ is the expected cost/revenue given that it deceeds the α-quantile (α is in the lower tail) of the cost/revenue and λ, $0 \leq \lambda \leq 1$, is a parameter indicating the risk attitude of the producer.

Generally the CVaR-term on for a confidence level α (with α in the left tail) is defined as

$$\mathrm{CVaR}_\alpha\left(X\right) = \eta_\alpha - \frac{1}{\alpha}\mathbb{E}\left[\max\left\{\left(\eta_\alpha - X\right), 0\right\}\right] \tag{9}$$

where η_α is the α-Value-at-Risk (VaR_α) or

$$\eta_\alpha = F_X^{-1}\left(\alpha\right) \tag{10}$$

where $F_X(\cdot)$ is the cumulative density function (CDF) of X.

If the prices and thereby the penalties are fixed we find that the CDF of the imbalance costs can be found as

$$\begin{aligned}
F_C(x) &= \mathbb{P}\left[C^{(\uparrow/\downarrow)} \leq x\right] \\
&= \mathbb{P}\left[\psi^{(\uparrow)}\left(W - \widetilde{W}\right) I\left\{W < \widetilde{W}\right\} - \psi^{(\downarrow)}\left(W - \widetilde{W}\right) I\left\{W > \widetilde{W}\right\} \leq x\right] \\
&= \mathbb{P}\left[W < \widetilde{W} \cap \psi^{(\uparrow)}\left(W - \widetilde{W}\right) \leq x\right] + \mathbb{P}\left[W > \widetilde{W} \cap -\psi^{(\downarrow)}\left(W - \widetilde{W}\right) \leq x\right] \\
&= \mathbb{P}\left[W \leq \widetilde{W} + \frac{x}{\psi^{(\uparrow)}}\right] + \mathbb{P}\left[W \geq \widetilde{W} - \frac{x}{\psi^{(\downarrow)}}\right] \\
&= F_W\left(\widetilde{W} + \frac{x}{\psi^{(\uparrow)}}\right) + 1 - F_W\left(\widetilde{W} - \frac{x}{\psi^{(\downarrow)}}\right)
\end{aligned} \tag{11}$$

which yields a PDF

$$f_C(x) = \frac{\partial}{\partial x} F_C(x) = \frac{1}{\psi^{(\uparrow)}} f_w\left(\frac{x}{\psi^{(\uparrow)}} + \widetilde{W}\right) + \frac{1}{\psi^{(\downarrow)}} f_w\left(-\frac{x}{\psi^{(\downarrow)}} + \widetilde{W}\right) \tag{12}$$

and thus

$$\begin{aligned}
\mathbb{E}\left[\max\left\{\left(\eta_\alpha - C^{(\uparrow/\downarrow)}\right), 0\right\}\right] = &\frac{1}{\psi^{(\uparrow)}} \int_{-\infty}^{\eta_\alpha} (\eta_\alpha - c) f_w\left(\frac{c}{\psi^{(\uparrow)}} + \widetilde{W}\right) dc \\
&+ \frac{1}{\psi^{(\downarrow)}} \int_{-\infty}^{\eta_\alpha} (\eta_\alpha - c) f_w\left(-\frac{c}{\psi^{(\downarrow)}} + \widetilde{W}\right) dc.
\end{aligned} \tag{13}$$

Now let

$$\begin{aligned}
c &= \psi^{(\uparrow)} \min\left\{W - \widetilde{W}, 0\right\} - \psi^{(\downarrow)} \max\left\{W - \widetilde{W}, 0\right\} \\
dc &= \left[\psi^{(\uparrow)} I\{\widetilde{W} > W\} - \psi^{(\downarrow)} I\{\widetilde{W} < W\}\right] dw
\end{aligned}$$

and obtain

$$
\begin{aligned}
&\mathbb{E}\left[\max\left\{\left(\eta_\alpha - C^{(\uparrow/\downarrow)}\right),0\right\}\right] \\
= \;& \frac{\psi^{(\uparrow)} I\{\widetilde{W} > W\} - \psi^{(\downarrow)} I\{\widetilde{W} < W\}}{\psi^{(\uparrow)}} \\
&\cdot \Bigg[\int\limits_{-\infty}^{\infty} \max\left\{\left(\eta_\alpha - \psi^{(\uparrow)} \min\left\{W-\widetilde{W},0\right\} + \psi^{(\downarrow)} \max\left\{W-\widetilde{W},0\right\}\right),0\right\} \\
&\qquad \cdot f_w\left(\frac{\psi^{(\uparrow)} \min\left\{W-\widetilde{W},0\right\} - \psi^{(\downarrow)} \max\left\{W-\widetilde{W},0\right\}}{\psi^{(\uparrow)}} + \widetilde{W}\right) dw\Bigg] \\
&+ \frac{\psi^{(\uparrow)} I\{\widetilde{W} > W\} - \psi^{(\downarrow)} I\{\widetilde{W} < W\}}{\psi^{(\downarrow)}} \\
&\cdot \Bigg[\int\limits_{-\infty}^{\infty} \max\left\{\left(\eta_\alpha - \psi^{(\uparrow)} \min\left\{W-\widetilde{W},0\right\} + \psi^{(\downarrow)} \max\left\{W-\widetilde{W},0\right\}\right),0\right\} \\
&\qquad \cdot f_w\left(-\frac{\psi^{(\uparrow)} \min\left\{W-\widetilde{W},0\right\} - \psi^{(\downarrow)} \max\left\{W-\widetilde{W},0\right\}}{\psi^{(\downarrow)}} + \widetilde{W}\right) dw\Bigg] \\
= \;& \frac{1}{\psi^{(\uparrow)}}\Bigg[\psi^{(\uparrow)} \int\limits_{0}^{\frac{\eta_\alpha + \psi^{(\uparrow)}\widetilde{W}}{\psi^{(\uparrow)}}} \left(\eta_\alpha - \psi^{(\uparrow)}\left(W-\widetilde{W}\right)\right) f_w\left(\frac{\psi^{(\uparrow)}\left(W-\widetilde{W}\right)}{\psi^{(\uparrow)}} + \widetilde{W}\right) dw \\
&+ \psi^{(\downarrow)} \int\limits_{\frac{\psi^{(\downarrow)}\widetilde{W} - \eta_\alpha}{\psi^{(\downarrow)}}}^{W^{(max)}} \left(\eta_\alpha + \psi^{(\downarrow)}\left(W-\widetilde{W}\right)\right) f_w\left(-\frac{\psi^{(\downarrow)}\left(W-\widetilde{W}\right)}{\psi^{(\uparrow)}} + \widetilde{W}\right) dw\Bigg] \qquad (14) \\
&+ \frac{1}{\psi^{(\downarrow)}}\Bigg[\psi^{(\uparrow)} \int\limits_{0}^{\frac{\eta_\alpha + \psi^{(\uparrow)}\widetilde{W}}{\psi^{(\uparrow)}}} \left(\eta_\alpha - \psi^{(\uparrow)}\left(W-\widetilde{W}\right)\right) f_w\left(\frac{\psi^{(\uparrow)}\left(W-\widetilde{W}\right)}{\psi^{(\downarrow)}} + \widetilde{W}\right) dw \\
&+ \psi^{(\downarrow)} \int\limits_{\frac{\psi^{(\downarrow)}\widetilde{W} - \eta_\alpha}{\psi^{(\downarrow)}}}^{W^{(max)}} \left(\eta_\alpha + \psi^{(\downarrow)}\left(W-\widetilde{W}\right)\right) f_w\left(-\frac{\psi^{(\downarrow)}\left(W-\widetilde{W}\right)}{\psi^{(\downarrow)}} + \widetilde{W}\right) dw\Bigg] \\
= \;& \int\limits_{0}^{\frac{\eta_\alpha + \psi^{(\uparrow)}\widetilde{W}}{\psi^{(\uparrow)}}} \Bigg[\left(\eta_\alpha - \psi^{(\uparrow)}\left(W-\widetilde{W}\right)\right) \cdot \\
&\left(f_w\left(W\right) + \frac{\psi^{(\uparrow)}}{\psi^{(\downarrow)}} f_w\left(\widetilde{W} - \frac{\psi^{(\uparrow)}}{\psi^{(\downarrow)}}\left(W-\widetilde{W}\right)\right)\right)\Bigg] dw \\
&+ \int\limits_{\frac{\psi^{(\downarrow)}\widetilde{W} - \eta_\alpha}{\psi^{(\downarrow)}}}^{W^{(max)}} \Bigg[\left(\eta_\alpha + \psi^{(\downarrow)}\left(W-\widetilde{W}\right)\right) \cdot \\
&\left(\frac{\psi^{(\downarrow)}}{\psi^{(\uparrow)}} f_w\left(\widetilde{W} - \frac{\psi^{(\downarrow)}}{\psi^{(\uparrow)}}\left(W-\widetilde{W}\right)\right) + f_w\left(W\right)\right)\Bigg] dw.
\end{aligned}
$$

Similarly for ρ we get

$$
\begin{aligned}
F_{\rho}(x) =&\ \mathbb{P}\left[\rho \leq x\right] \\
=&\ \mathbb{P}\left[W < \widetilde{W} \ \&\ \pi^{(S)}\widetilde{W} + \pi^{(\uparrow)}\left(W - \widetilde{W}\right) \leq x\right] \\
&+ \mathbb{P}\left[W > \widetilde{W} \ \&\ \pi^{(S)}\widetilde{W} + \pi^{(\downarrow)}\left(W - \widetilde{W}\right) \leq x\right] \\
=&\ \mathbb{P}\left[W \leq \frac{x}{\pi^{(\uparrow)}} + \widetilde{W}\left(1 - \frac{\pi^{(S)}}{\pi^{(\uparrow)}}\right)\right] + \mathbb{P}\left[\widetilde{W} \leq W \leq \frac{x}{\pi^{(\downarrow)}} + \widetilde{W}\left(1 - \frac{\pi^{(S)}}{\pi^{(\downarrow)}}\right)\right] \\
=&\ F_W\left(\frac{x}{\pi^{(\uparrow)}} + \widetilde{W}\left(1 - \frac{\pi^{(S)}}{\pi^{(\uparrow)}}\right)\right) + F_W\left(\frac{x}{\pi^{(\downarrow)}} + \widetilde{W}\left(1 - \frac{\pi^{(S)}}{\pi^{(\downarrow)}}\right)\right) - F_W\left(\widetilde{W}\right)
\end{aligned}
\tag{15}
$$

thus yielding

$$
\begin{aligned}
f_{\rho}(x) &= \frac{\partial}{\partial x} F_{\rho}(x) \\
&= \frac{1}{\pi^{(\uparrow)}} f_W\left(\frac{x}{\pi^{(\uparrow)}} + \widetilde{W}\left(1 - \frac{\pi^{(S)}}{\pi^{(\uparrow)}}\right)\right) + \frac{1}{\pi^{(\downarrow)}} f_W\left(\frac{x}{\pi^{(\downarrow)}} + \widetilde{W}\left(1 - \frac{\pi^{(S)}}{\pi^{(\downarrow)}}\right)\right)
\end{aligned}
\tag{16}
$$

which in turn renders

$$
\begin{aligned}
\mathbb{E}\left[\max\left\{(\eta_{\alpha} - \rho), 0\right\}\right] = &\ \frac{1}{\pi^{(\uparrow)}} \int_{-\infty}^{\eta_{\alpha}} (\eta_{\alpha} - r)\, f_w\left(\frac{r}{\pi^{(\uparrow)}} + \widetilde{W}\left(1 - \frac{\pi^{(S)}}{\pi^{(\uparrow)}}\right)\right) dr \\
&+ \frac{1}{\pi^{(\downarrow)}} \int_{-\infty}^{\eta_{\alpha}} (\eta_{\alpha} - r)\, f_w\left(\frac{r}{\pi^{(\downarrow)}} + \widetilde{W}\left(1 - \frac{\pi^{(S)}}{\pi^{(\downarrow)}}\right)\right) dr.
\end{aligned}
\tag{17}
$$

Following the same procedure as before and let

$$
\begin{aligned}
r &= \pi^{(S)}\widetilde{W} + \pi^{(\uparrow)} \min\left\{W - \widetilde{W}, 0\right\} + \pi^{(\downarrow)} \max\left\{W - \widetilde{W}, 0\right\} \\
dr &= \left[\psi^{(\uparrow)} I\left\{\widetilde{W} > W\right\} - \psi^{(\downarrow)} I\left\{\widetilde{W} < W\right\}\right] dw
\end{aligned}
$$

and thereby obtain

$$
\begin{aligned}
&\mathbb{E}\left[\max\left\{\left(\eta_\alpha-\rho\right),0\right\}\right]\\
=\;&\frac{\psi^{(\uparrow)}I\left\{\widetilde{W}>W\right\}-\psi^{(\downarrow)}I\left\{\widetilde{W}<W\right\}}{\pi^{(\uparrow)}}\\
&\cdot\left[\int\limits_{-\infty}^{\infty}\max\left\{\left(\eta_\alpha-\pi^{(S)}\widetilde{W}-\pi^{(\uparrow)}\min\left\{W-\widetilde{W},0\right\}-\pi^{(\downarrow)}\max\left\{W-\widetilde{W},0\right\}\right),0\right\}\right.\\
&\qquad\cdot f_w\left(\frac{\pi^{(S)}\widetilde{W}+\pi^{(\uparrow)}\min\left\{W-\widetilde{W},0\right\}+\pi^{(\downarrow)}\max\left\{W-\widetilde{W},0\right\}}{\pi^{(\uparrow)}}\right.\\
&\qquad\qquad\left.\left.+\widetilde{W}\left(1-\frac{\pi^{(S)}}{\pi^{(\uparrow)}}\right)\right)dw\right]\\
&+\frac{\psi^{(\uparrow)}I\left\{\widetilde{W}>W\right\}-\psi^{(\downarrow)}I\left\{\widetilde{W}<W\right\}}{\pi^{(\downarrow)}}\\
&\cdot\left[\int\limits_{-\infty}^{\infty}\max\left\{\left(\eta_\alpha-\pi^{(S)}\widetilde{W}-\pi^{(\uparrow)}\min\left\{W-\widetilde{W},0\right\}-\pi^{(\downarrow)}\max\left\{W-\widetilde{W},0\right\}\right),0\right\}\cdot\right.\\
&\qquad\cdot f_w\left(\frac{\pi^{(S)}\widetilde{W}+\pi^{(\uparrow)}\min\left\{W-\widetilde{W},0\right\}+\pi^{(\downarrow)}\max\left\{W-\widetilde{W},0\right\}}{\pi^{(\downarrow)}}\right.\\
&\qquad\qquad\left.\left.+\widetilde{W}\left(1-\frac{\pi^{(S)}}{\pi^{(\downarrow)}}\right)\right)dw\right]\\
=\;&\int\limits_{0}^{\frac{\eta_\alpha+\widetilde{W}\left(\pi^{(\uparrow)}-\pi^{(S)}\right)}{\pi^{(\uparrow)}}}\left[\left(\eta_\alpha-\pi^{(S)}\widetilde{W}-\pi^{(\uparrow)}\left(W-\widetilde{W}\right)\right)\cdot\right.\\
&\qquad\left.\left(\frac{\psi^{(\uparrow)}}{\pi^{(\uparrow)}}f_w\left(W\right)+\frac{\psi^{(\uparrow)}}{\pi^{(\downarrow)}}f_w\left(\widetilde{W}+\frac{\pi^{(\uparrow)}}{\pi^{(\downarrow)}}\left(W-\widetilde{W}\right)\right)\right)\right]dw\\
&+\int\limits_{\frac{\eta_\alpha-\widetilde{W}\left(\pi^{(S)}-\pi^{(\downarrow)}\right)}{\pi^{(\downarrow)}}}^{W^{(max)}}\left[\left(\eta_\alpha-\pi^{(S)}\widetilde{W}-\pi^{(\downarrow)}\left(W-\widetilde{W}\right)\right)\cdot\right.\\
&\qquad\left.\left(\frac{\psi^{(\downarrow)}}{\pi^{(\uparrow)}}f_w\left(\widetilde{W}+\frac{\pi^{(\downarrow)}}{\pi^{(\uparrow)}}\left(W-\widetilde{W}\right)\right)+\frac{\psi^{(\downarrow)}}{\pi^{(\downarrow)}}f_w\left(W\right)\right)\right]dw.
\end{aligned}
\tag{18}
$$

Considering the prices to be fixed, as is done in Eqs. (14) and (18), allows for a closed form solution of the problem to be obtained (Dent et al. (2011)). The prices are however stochastic and as shown below the price expectation integrals can not be separated from the wind expectation integral. Thus the certainty equivalence principle does not apply anymore.

With stochastic prices, Eq. (14) & (18) become

$$
\begin{aligned}
&\mathbb{E}\left[\max\left\{\left(\eta_\alpha\left(\boldsymbol{\psi}\right)-C^{(\uparrow/\downarrow)}\right),0\right\}\right]\\
=&\int_0^\infty\int_0^\infty\int_0^{\frac{\eta_\alpha(\boldsymbol{\psi})+\psi^{(\uparrow)}\widetilde{W}}{\psi^{(\uparrow)}}}\left(\eta_\alpha\left(\boldsymbol{\psi}\right)-\psi^{(\uparrow)}\left(W-\widetilde{W}\right)\right)\\
&\cdot\left(f_w\left(W\right)+\frac{\psi^{(\uparrow)}}{\psi^{(\downarrow)}}f_w\left(\widetilde{W}-\frac{\psi^{(\uparrow)}}{\psi^{(\downarrow)}}\left(W-\widetilde{W}\right)\right)\right)f_{\boldsymbol{\psi}}(\boldsymbol{\psi})dwd\boldsymbol{\psi}\\
&+\int_0^\infty\int_0^\infty\int_{\frac{\psi^{(\downarrow)}\widetilde{W}-\eta_\alpha(\boldsymbol{\psi})}{\psi^{(\downarrow)}}}^{W^{(max)}}\left(\eta_\alpha\left(\boldsymbol{\psi}\right)+\psi^{(\downarrow)}\left(W-\widetilde{W}\right)\right)\\
&\cdot\left(\frac{\psi^{(\downarrow)}}{\psi^{(\uparrow)}}f_w\left(\widetilde{W}-\frac{\psi^{(\downarrow)}}{\psi^{(\uparrow)}}\left(W-\widetilde{W}\right)\right)+f_w\left(W\right)\right)f_{\boldsymbol{\psi}}(\boldsymbol{\psi})dwd\boldsymbol{\psi}.
\end{aligned}
\tag{19}
$$

and

$$
\begin{aligned}
&\mathbb{E}\left[\max\left\{\left(\eta_\alpha\left(\boldsymbol{\pi}\right)-\rho\right),0\right\}\right]\\
=&\int_{-\infty}^{\infty}\int_{\pi^{(S)}}^{\infty}\int_{-\infty}^{\pi^{(S)}}\int_0^{\frac{\eta_\alpha(\boldsymbol{\pi})+\widetilde{W}\left(\pi^{(\uparrow)}-\pi^{(S)}\right)}{\pi^{(\uparrow)}}}\left(\eta_\alpha\left(\boldsymbol{\pi}\right)-\pi^{(S)}\widetilde{W}-\pi^{(\uparrow)}\left(W-\widetilde{W}\right)\right)\\
&\cdot\left(\frac{\psi^{(\uparrow)}}{\pi^{(\uparrow)}}f_w\left(W\right)+\frac{\psi^{(\uparrow)}}{\pi^{(\downarrow)}}f_w\left(\widetilde{W}+\frac{\pi^{(\uparrow)}}{\pi^{(\downarrow)}}\left(W-\widetilde{W}\right)\right)\right)f_{\boldsymbol{\pi}}(\boldsymbol{\pi})dwd\boldsymbol{\pi}\\
&+\int_{-\infty}^{\infty}\int_{\pi^{(S)}}^{\infty}\int_{-\infty}^{\pi^{(S)}}\int_{\frac{\eta_\alpha(\boldsymbol{\pi})-\widetilde{W}\left(\pi^{(S)}-\pi^{(\downarrow)}\right)}{\pi^{(\downarrow)}}}^{W^{(max)}}\left(\eta_\alpha\left(\boldsymbol{\pi}\right)-\pi^{(S)}\widetilde{W}-\pi^{(\downarrow)}\left(W-\widetilde{W}\right)\right)\\
&\cdot\left(\frac{\psi^{(\downarrow)}}{\pi^{(\uparrow)}}f_w\left(\widetilde{W}+\frac{\pi^{(\downarrow)}}{\pi^{(\uparrow)}}\left(W-\widetilde{W}\right)\right)+\frac{\psi^{(\downarrow)}}{\pi^{(\downarrow)}}f_w\left(W\right)\right)f_{\boldsymbol{\pi}}(\boldsymbol{\pi})dwd\boldsymbol{\pi}
\end{aligned}
\tag{20}
$$

respectively where

$$
\begin{aligned}
\boldsymbol{\psi}&=\left[\begin{array}{cc}\psi^{(\downarrow)} & \psi^{(\uparrow)}\end{array}\right]\\
\boldsymbol{\pi}&=\left[\begin{array}{ccc}\pi^{(\downarrow)} & \pi^{(\uparrow)} & \pi^{(S)}\end{array}\right]
\end{aligned}
$$

are multivariate stochastic variables with the appropriate covariance structure.

What is still missing in the expressions are the formulations of $\eta_\alpha(\cdot)$ in terms of the wind and the price/penalty variables. Assuming that both the penalty and the imbalance had a parametric distribution and were independent of each other an analytical expression for the variance of the cost can be found (in terms of $\psi^{(\uparrow/\downarrow)}$) as

$$
\mathbb{V}\left[\psi(W-\widetilde{W})\right]=\mathbb{V}\left[\psi\right]\left(\mathbb{E}\left[(W-\widetilde{W})\right]\right)^2+\mathbb{V}\left[(W-\widetilde{W})\right]\left(\mathbb{E}\left[\psi\right]\right)^2+\mathbb{V}\left[\psi\right]\mathbb{V}\left[(W-\widetilde{W})\right] \tag{21}
$$

from which η_α could be derived. The distribution of W is however known to be best characterised by non-parametric techniques and the same probably goes for the imbalance penalties/prices (although probably nothing should be ruled out in this regard). More importantly though $\psi^{(\uparrow/\downarrow)}$ and $(W-\widetilde{W})$ are not necessarily independent. Even though η_α could be found with the help of Eq. (21), an analytical solution to the optimisation problem would be hard, if not impossible, to obtain. Thus a discrete formulation of the problem using scenarios of the cost is probably a more likely road to success.

4 Operational Risk-Averse Bidding

4.1 Bidding as a Price Taker

Now assume that a set of N cost/revenue scenarios, ζ_s, $s \in \mathcal{S}$, is available and that they are ordered such that $\zeta_1 < \zeta_2 < \ldots < \zeta_N$. Then by defining the index k_α such that

$$\sum_{k=1}^{k_\alpha} p_k \leq \alpha < \sum_{k=1}^{k_\alpha+1} p_k,$$

where p_s is the probability of scenario s, the mean-risk bidding optimisation problem can be written as

$$\begin{aligned}
\widetilde{W}_t^{(*)} &= \arg\max_{\widetilde{W}_t} \left\{ (1-\lambda)\mathbb{E}\left[\zeta(\widetilde{W})\right] + \lambda \mathrm{CVaR}_\alpha \left[\zeta(\widetilde{W})\right] \right\} \\
&= \arg\max_{\widetilde{W}_t} \left\{ (1-\lambda) \sum_{s \in \mathcal{S}} p_s \zeta_s(\widetilde{W}) \right. \\
&\left. + \lambda \left(\eta_\alpha(\widetilde{W}) - \frac{1}{\alpha} \left[\sum_{k=1}^{k_\alpha} p_k \zeta_k(\widetilde{W}) + \left(\sum_{k=k_\alpha+1}^{N} p_k - (1-\alpha) \right) \zeta_{k_\alpha}(\widetilde{W}) \right] \right) \right\}
\end{aligned} \tag{22}$$

where, depending on whether the objective is formulated in terms of cost or revenue, $\zeta_s(\widetilde{W})$ is either defined as

$$\zeta_s(\widetilde{W}) = I\left\{ W_s > \widetilde{W} \right\} (W_s - \widetilde{W})\psi_s^{(\downarrow)} + I\left\{ W_s < \widetilde{W} \right\} (W_s - \widetilde{W})\psi_s^{(\uparrow)} \tag{23}$$

or

$$\zeta_s(\widetilde{W}) = \pi_s^{(S)} \widetilde{W}_s + I\left\{ W_s > \widetilde{W} \right\} (W_s - \widetilde{W})\pi_s^{(\downarrow)} + I\left\{ W_s < \widetilde{W} \right\} (W_s - \widetilde{W})\pi_s^{(\uparrow)} \tag{24}$$

respectively. As before $\eta_\alpha(\widetilde{W})$ is the α-VaR or

$$\eta_\alpha(\widetilde{W}) = \mathrm{VaR}_\alpha(\widetilde{W}) = F^{-1}_{\zeta(\widetilde{W})}(\alpha)$$

Finally the producer's risk attitude is characterised by the parameter λ, $0 \leq \lambda \leq 1$, which takes the value 0 for the completely risk-neutral producer and the value 1 for total risk-aversion.

Now rewrite Eq. (22) as

$$\widetilde{W}_t^{(*)} = \arg\max_{\widetilde{W}} \left\{ (1-\lambda) \sum_{s \in \mathcal{S}} p_s \zeta_s(\widetilde{W}) + \lambda \left(\eta_\alpha(\widetilde{W}) - \frac{1}{\alpha} \sum_{s \in \mathcal{S}} p_s \xi_s(\widetilde{W}) \right) \right\} \tag{25}$$

$$s.t. \quad \xi_s(\widetilde{W}) \geq \eta_\alpha(\widetilde{W}) - \zeta_s(\widetilde{W}) \tag{26}$$

$$\xi_s(\widetilde{W}) \geq 0 \tag{27}$$

and introduce two auxiliary variables

$$\Delta W_s^{(\uparrow)}(\widetilde{W}) \leq W_s - \widetilde{W} \tag{28}$$

$$\Delta W_s^{(\downarrow)}(\widetilde{W}) \leq \widetilde{W} - W_s \tag{29}$$

$$\Delta W_s^{(\uparrow)}(\widetilde{W}), \Delta W_s^{(\downarrow)}(\widetilde{W}) \leq 0. \tag{30}$$

Consequently Eq. (23) & (24) can be written as

$$\zeta_s(\widetilde{W}) = \Delta W_s^{(\downarrow)}(\widetilde{W})\psi_s^{(\downarrow)} + \Delta W_s^{(\uparrow)}(\widetilde{W})\psi_s^{(\uparrow)} \tag{31}$$

and

$$\begin{aligned}\zeta_s(\widetilde{W}) &= \pi_s^{(S)}\widetilde{W} + \Delta W_s^{(\downarrow)}(\widetilde{W})\pi_s^{(\downarrow)} + \Delta W_s^{(\uparrow)}(\widetilde{W})\pi_s^{(\uparrow)} \\ &= \pi_s^{(S)}W_s + \Delta W_s^{(\downarrow)}(\widetilde{W})\psi_s^{(\downarrow)} + \Delta W_s^{(\uparrow)}(\widetilde{W})\psi_s^{(\uparrow)}\end{aligned} \tag{32}$$

respectively. The augmented optimisation problem (25) subject to (26) - (30) can be solved, for given values of α and λ, as an LP problem with a maximum

$$\left(\widetilde{W}^{(*)}, \eta_\alpha^{(*)}(\widetilde{W}^{(*)}), \Delta W^{(\uparrow,*)}(\widetilde{W}^{(*)}), \Delta W^{(\downarrow,*)}(\widetilde{W}^{(*)})\right)$$

for which it holds that

$$\eta_\alpha^{(*)}(\widetilde{W}^{(*)}) = \mathrm{VaR}_\alpha\left[\zeta(\widetilde{W}^{(*)})\right] \tag{33}$$

$$\eta_\alpha^{(*)}(\widetilde{W}^{(*)}) - \frac{1}{\alpha}\sum_{s\in\mathcal{S}} p_s\xi_s(\widetilde{W}) = \mathrm{CVaR}_\alpha\left[\zeta(\widetilde{W}^{(*)})\right] \tag{34}$$

thus making $\widetilde{W}^{(*)}$ also the optimal solution of (22).

4.2 Bidding as a Price Maker

Abandoning the price taker assumption can be done in two steps. First by assuming that the producer is only a price taker on the day-ahead market (where there are more producers and volume involved) but is a price maker or has some sort of market power on the regulation market. Subsequently the the producer can be assumed to possess market power on both markets. Whether the possibility of being a price maker only on the regulation market is realistic will not be addressed in detail for the time being. However given that there is less volume "traded" on the regulation market and that there are fewer potential actors on that market, this scenario seems intuitively plausible.

The main difference between solving the bidding problem for a price maker and a price taker is that the scenarios (or the scenario probabilities) now depend on the decision. Essentially we are still interested in solving the problem given in Eq. (25) only with the decision dependent scenarios. If we discretise the decision space such that we have a set, $\mathcal{W}$, of finite number of decisions intervals and for each $w \in \mathcal{W}$ we have a corresponding set, $\mathcal{S}^w$ of cost/revenue scenarios that have a constant probabilities on the entire span of w. Then the problem in (25) can be revised to

$$\widetilde{W}_t^{(*)} = \arg\max_{w\in\mathcal{W}} C^w\left(w;\alpha,\lambda\right) \tag{35}$$

where

$$C^w\left(w;\alpha,\lambda\right) = \max\left\{(1-\lambda)\sum_{s\in\mathcal{S}^w} p_s\zeta_s(\widetilde{W}) + \lambda\left(\eta_\alpha(\widetilde{W}) - \frac{1}{\alpha}\sum_{s\in\mathcal{S}^w} p_s\xi_s(\widetilde{W})\right)\right\} \tag{36}$$

still subject to (26)-(30) and with $\zeta(\widetilde{W})$ defined by (31) or (32).

Given a set $\mathcal{W}$ containing N_w decisions and for each $w \in \mathcal{W}$ a corresponding set of N_s cost/revenue scenarios, the two-stage problem in (35) & (36) can be solved as is. This will however of course require generation of $N_w \times N_s$ scenarios in total which rather quickly becomes a substantial amount.

An alternative to generating N_w different scenario sets, is only to generate a single set of cost/revenue scenarios and let the scenario probabilities, $p_{s,w}$, vary between different w's. Generally this is possible if zero scenario probabilities are allowed. However as the variations in the bid won't in any case alter the range of possible outcomes, this can be done for this particular case without having to assign zero scenario probabilities. So again let $\mathcal{W}$ be a set of bidding decisions, $\mathcal{S}$ be a set of cost/revenue scenarios and $\boldsymbol{P}_s$ be an $N_s \times N_w$ matrix of scenario probabilities, with element (s, w) stating the probability of scenario s under decision w. Furthermore let $\widetilde{w}_i$ be a bidding decision on the interval $w_i \in \mathcal{W}$. Finally define a (column) vector of binary variables $\boldsymbol{z}$ of length N_w and with elements z_{w_i}. Then then we can write the bidding problem as a mixed integer problem:

$$\widetilde{W}_t^{(*)} = \arg\max_{\widetilde{W}} \left\{ (1-\lambda) \sum_{s \in \mathcal{S}} p_s \zeta_s(\widetilde{W}) + \lambda \left(\eta_\alpha - \frac{1}{\alpha} \sum_{s \in \mathcal{S}} p_s \xi_s(\widetilde{W}) \right) \right\} \tag{37}$$

$$s.t. \quad \xi_s(\widetilde{W}) \geq \eta_\alpha(\widetilde{W}) - \zeta_s(\widetilde{W}) \tag{38}$$

$$\xi_s(\widetilde{W}) \geq 0 \tag{39}$$

$$\Delta W_s^{(\uparrow)}(\widetilde{W}) \leq W_s - \widetilde{W} \tag{40}$$

$$\Delta W_s^{(\downarrow)}(\widetilde{W}) \leq \widetilde{W} - W_s \tag{41}$$

$$\Delta W_s^{(\uparrow)}(\widetilde{W}), \Delta W_s^{(\downarrow)}(\widetilde{W}) \leq 0 \tag{42}$$

$$z_{w_i} \underline{w}_i \leq \widetilde{w}_i \leq z_{w_i} \overline{w}_i \tag{43}$$

$$\sum_{w_i \in \mathcal{W}} z_{w_i} = 1 \tag{44}$$

$$\widetilde{W} = \sum_{i=1}^{N_w} w_i \tag{45}$$

$$p_s = \sum_{i=1}^{N_w} \boldsymbol{P}_s(s, i) z_{w_i} = \boldsymbol{P}_s(s, :) \boldsymbol{z} \tag{46}$$

where $\underline{w}_i$ and $\overline{w}_i$ are the lower and the upper bound of w_i respectively and $\zeta(\widetilde{W})$ still defined by (31) or (32). So given the necessary scenarios, this mixed integer formulation of the problem can then be used to optimise the bids of a producer that only has market power on the regulation market as well as for a producer that also has market power on the day-ahead market. Whereas in the former case, one still has the choice between optimising in terms of his imbalance costs or in therm of his revenues, the revenue formulation is probably the only sensible formulation for a producer with full market power. This is because the first one is still not in any control over his spot market revenues and thus can look away from them in the optimisation. In contrast, the second one can diminish the value of his lower imbalance cost by reducing his spot market revenues in the process.

4.3 Objective - Revenue vs. Imbalance Costs

The choice between the two possible formulations of the objective function is not obvious. In Zugno et al. (2012) the cost formulation is chosen since it can be shown that maximising the expected deviation from perfect information revenue (or the expected imbalance costs) the same as maximising the expected revenue for a price taker. Once any nonlinear measure of risk becomes an explicit part of the objective however this is no longer true - Regardless of whether you only consider production risk or you consider the production and the price risk together. This in turn yields different optimums depending on the formulation chosen. Summarised in one sentence the difference for a mixed strategy ($0 < \lambda < 1$) is that whereas the CVaR for the cost formulation pulls the bid towards the expected production, the CVaR for the revenue pulls the bid towards the bid yielding the lowest probability of negative revenue.

In order to further illustrate the difference between the two consider the following toy example. Assume that we have for a given hour 10 equally probable scenarios of wind power production so

$$\begin{bmatrix} w_{s_1} & w_{s_2} & \dots & w_{s_{10}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & \dots & 9 \end{bmatrix} MWh/h$$
$$p_i = 0.1 \quad \forall i = 1, 2, \dots, 10.$$

Furthermore assume that up-and down-regulation are equally probable for that same hour and that the market prices (conditioned upon their realisation where appropriate) are fixed at

$$\pi^{(S)} = 20, \quad \pi^{(\uparrow)} = 30, \pi^{(\downarrow)} = 10 \Rightarrow \psi^{(\uparrow)} = \psi^{(\downarrow)} = 10.$$

Finally let $\alpha = 0.1$ and assume that wind power is always produced, i.e. that what's not sold day-ahead is sold at the down regulation price. Given all this, the bid maximising both the expected revenues and the (non-positive) expected imbalance cost is the expected production or $4.5MWh/h$. The CVaR however is maximised at $0MWh/h$ for the revenue formulation but at $4.5MWh/h$ for the cost formulation.

What makes the maximum CVaR be at zero for the revenue formulation is that since the prices are fixed such that the down regulation price is positive, a non-negative revenue is guaranteed by bidding zero into the market. Then making the prices stochastic and the probability of negative down regulation price positive pushes the bid again away from zero - still contributing to a lower bid than a pure optimisation of the expected revenue would though. The push towards the expected production in the other case however is a result of the fact that unless very different probabilities of each imbalance sign the worst case scenarios are to be found at both ends of the revenue distribution. Hence the push towards the expected value.

Which of the two formulations is the more appropriate one is to some extent a matter of preference and there are arguments in favour of either one. For one the producer's financial health is in the end affected by his revenues and therefore one can argue that revenue risks should be considered when power is bid to the market. This was also the initial objective of the strategy in Zugno et al. (2012) which was only revised since the cost formulation was equivalent. For a risk-averse producer, an additional appeal of the revenue formulation is that it offers the bid that truly minimises the probability of loosing money. Hence the revenue formulation can be said to be more conservative than the cost formulation. However it is important for the system operation standpoint that the volume bid to the market reflects the actual future production. This prompted the constraint that the bid should be within a certain limit from the point forecast in Zugno et al. (2012) and for the revenue formulation of the CVaR bidding strategy, this constraint is still likely to be necessary in the same form as before. In contrast optimising the CVaR in the cost formulation essentially has the same effect as that constraint. Thus with a carefully chosen value of λ will in some ways account for this constraint implicitly and what is more do it in a more intelligent manner. In addition, the coherence of CVaR implies that

$$\mathrm{CVaR}_\alpha[\rho] \leq \mathrm{CVaR}_\alpha[\rho^{(S)}] + \mathrm{CVaR}_\alpha[\rho^{(\uparrow/\downarrow)}]$$

where

$$\rho^{(S)} = \pi_s^{(S)} W_s$$
$$\rho^{(\uparrow/\downarrow)} = \Delta W_s^{(\downarrow)}(\widetilde{W})\psi_s^{(\downarrow)} + \Delta W_s^{(\uparrow)}(\widetilde{W})\psi_s^{(\uparrow)}$$

and $\rho^{(S)}$ consists of two stochastic variables, neither of which the is in control of. So one can argue that he/she who actually is responsible for trading wind power should focus on maintaining as little deviation as possible from the perfect scenario and let someone else take care of the revenue risk as such - possibly along with the producer's total revenue risk if it has a diverse production portfolio. So in addition to general taste, the appropriate formulation might also depend on how the general structure of the company owning the turbines is. For a producer only owning wind turbines the revenue formulation might be more appropriate while the cost formulation might be better suited for a large company with a diverged production portfolio. For both objectives it is important to realise though that long-term goals for risk can be derived directly from the hourly risk criteria. Due to the lack of apparent better choice it is decided to try out both formulation in attempt to shed more light on the consequences of adopting either formulation of the strategy.

5 Scenario Generation

In order to solve the problem in the formulation just described, scenarios for production and market outcomes are necessary. In this section generation of such scenarios are loosely discussed. First the method used to obtain the scenarios used in the case study later presented are discussed. Thereafter, some thoughts on how these scenarios could be constructed properly are presented.

5.1 Simple Scenarios

In order to find the optimal bid in terms of revenue, each scenario $s \in \mathcal{S}$ consists of simulated realisations of 4 different variables. These are the wind power production, W_s, the spot price, $\pi_s^{(S)}$, and the up-and down regulation penalties, $\psi_s^{(\uparrow)}$ & $\psi_s^{(\downarrow)}$. For the cost formulation, the same variables are needed apart from the spot price. For now (and in the price taker test case presented in the next section) we'll assume that there isn't any type of dependence between the wind power production and the prices so that wind power scenarios and market scenarios can be created independently of each other. Doing so allows us to create a wind scenario only by drawing random realisations of the estimated CDF for that particular time. For the market, its sequential arrangement prompts a hierarchy in the prices which somehow has to be taken into account when market scenarios are generated. Therefore it is decided to model the density of the penalties conditional to the actual spot price and adopt the following scheme for generating the scenarios (including the wind power):

For generating each $s \in \mathcal{S}$:

1. Generate a vector $\boldsymbol{r}$ with IID elements $r_i \sim U(0,1)\forall i$, $i = 1, \ldots, 4$
2. Set $\pi_s^{(S)} = F_{\pi^{(S)}}^{-1}(r_1)$
3. Set $I_s^{(\psi)} = I(\psi^{(\uparrow)} > 0) - I(\psi^{(\downarrow)} < 0) = F_I^{-1}(r_2)$

4. Set $W_s = F_W^{-1}(r_3)$
5. If $I_s^{(\psi)} = -1$

   I Estimate $F_{\psi^{(\downarrow)}}(x; \pi^{(S)}, I_s^{(\psi)})$

   II Set $\psi_s^{(\downarrow)} = F_{\psi^{(\downarrow)}}^{-1}(r_4; \pi^{(S)}, I_\psi)$

   III Set $\psi_s^{(\uparrow)} = 0$

   else if $I_s^{(\psi)} = 1$

   I Estimate $F_{\psi^{(\uparrow)}}(x; \pi_s^{(S)}, I_s^{(\psi)})$

   II Set $\psi_s^{(\uparrow)} = F_{\psi^{(\uparrow)}}^{-1}(r_4; \pi_s^{(S)}, I_s^{(\psi)})$

   III Set $\psi_s^{(\downarrow)} = 0$

   else $\psi_s^{(\uparrow)} = \psi_s^{(\downarrow)} = 0$

For the cost formulation, the spot prices are not a part of the scenarios and thus unconditional densities for the penalties are required. However since

$$F_\psi(x) = \int_{-\infty}^{\infty} F_\psi(x; \pi^{(S)}) d\pi^{(S)}$$

by generating a large number of scenarios as described above and discarding the spot prices is a completely valid alternative to constructing new densities.

Regarding forecasts, those are required for the following variables:

1. The day-ahead spot price, $\pi^{(S)}$
2. The wind power production, W
3. The imbalance sign, $I_s^{(\psi)} = I(\psi^{(\uparrow)} > 0) - I(\psi^{(\downarrow)} < 0)$
4. The regulation penalties, conditioned on it being different from zero, i.e. $\psi^{(\uparrow)} | I^{(\psi)} = 1$ and $\psi^{(\downarrow)} | I^{(\psi)} = -1$

5.2 Further Development

The scenario generation scheme listed before is complete in the sense that all realisations all necessary variables are created based obtainable predictive densities. Moreover, should the assumption that there is no relationship between our wind power production and the market outcome holds, the method for scenario generation outlined above complete. Whereas it's difficult to see any reason for this assumption not to hold for the spot market, since it is settled before the production is realised, the situation is more complicated for the regulation market. First of all, unless the producer's turbines are relatively isolated geographically, the production of the turbines and the remaining turbines in the system is going to be positively correlated. Then if this correlation is present, the question becomes whether or not the actual production in the system affects the market outcome or not. Now the imbalance sign (defined by price) is determined by the system imbalance (and whether or not the prompt a penalty) which in turn intuitively seems influenced by that of the wind turbines - At least during hours of medium to high production.

As previously mentioned, the relationship between the wind power production and regulation penalty is two sided. For one, increased offering of wind power in the spot market prompts a greater supply of relatively cheap regulating power since flexible units otherwise cleared on the spot market will now be offering regulative power. The increased share of wind power however also typically prompts a greater demand for balancing power which then counteracts with the increased supply. In addition, the generation units capable of supplying balancing power all operate with some substantial marginal cost. Therefore even if you only look at the supply side there is a limit on how low the regulation price can be. Thus at some point the increased share of wind in the cleared volume starts to increase the penalty again. Empirical evidence supporting these statements can be viewed in Jónsson et al. (2012a).

So for the purpose of better scenario generation, both for a price taker and a price maker, it would be interesting to first model a joint distribution between one's own production errors and those of the system and subsequently joint that distribution with the regulation market densities e.g. through copulas by the method outlined in Pinson et al. (2009). Although easier said than done, this task should be well within what's achievable.

On a more overall note, there are several other factors that impact the regulation prices which impact the regulation market outcome. An example of two of the more important ones of these factors are the demand side imbalance and to which degree exporting/importing the imbalances to neighbouring areas exists, if at all. These variables are of course not a part of the required scenario explicitly. Basing the regulation market scenarios on that for these extra variables however could help towards gaining a more accurate description of the probability density of the imbalance penalties.

Once models for the aforementioned joint densities have been developed, the scenario reduction technique from Nicole Growe-Kuska and Romisch (2003) could be adopted to create scenarios for the price maker solution. In short, the forward selection method of Nicole Growe-Kuska and Romisch (2003) calculates the euclidean distances between every scenario in a given set of scenarios, selects a reduced number of scenarios that conserve the moments of the original set and reassigns scenario probabilities. So given a number of different scenarios sets for different wind power bid to the market, each of these scenarios sets could be reduced to the same pre-determined scenarios that match the unconditional revenue distribution.

6 A (Really) Small Case Study

Finally we present a small case study to illustrate the characteristics of the bidding behaviour of a producer adopting the strategy here described. Before the actual bidding results are presented, a brief discussion of the forecasts constructed for the purpose is given.

6.1 Forecasts

For all variables, the procedure for obtaining the forecasts for all the continuous variables is similar. First a point forecast is made and then prediction intervals are estimated using methods that explicitly forecast each quantile for a range of pre-determined set of quantiles.

All models are tuned on data for the period from November 1st 2008 - January 31st 2010.

The point forecasts are obtained in the following manner:

- The wind power forecasts stem from WPPT, a statistically based and commercially based prediction softwareNielsen et al. (2002) (see `www.enfor.dk` for further information)
- The spot price predictions are found using the model presented in Jónsson et al. (2011).
- The imbalance penalty forecasts are found by a Holt-Winters model similar to the one described in Jónsson et al. (2012a).

For all variables, the time-adaptive quantile regression method described in Møller et al. (2008) and Jónsson et al. (2012b) are used to predict density quantiles of nominal proportion between 0.05 and 0.95 in steps of 0.05. For the wind power no further modelling is done and the 0 and installed capacity set as the 0 and 100% quantiles. For the prices and penalties, the Conditional Autoregressive Value-at-Risk (CAViaR)Gorr and Hsu (1985); Engle and Manganelli (2004); Taylor (2008) model are used to predict the $0.01, 0.025, 0.975, 0.99$ quantiles as well. Then the highest and the lowest observations during the data period are set as the 0 and 100% quantiles.

Finally, the imbalance sign probabilities are obtained by the Holt-Winters model described in Jónsson et al. (2012a).

6.2 Bidding Behaviour

In order to illustrate the workings of the bidding strategy and the different formulations of the objective, the strategy is tested for a single hour. The hour chosen is the 10th hour of July 13th 2009 for which we have the forecasts shown in Figure 1. These forecasts are then used to generate 10000 scenarios for the all variables involved using the previously described scenario generation procedure and subsequently reduced to 100 by the method of Nicole Growe-Kuska and Romisch (2003). After that the optimal bid is calculated using GAMS for $\alpha = 0.05$ and $\alpha = 0.1$ and the value of λ is varied between 0 and 1 in steps of 0.1 and compared to the bids obtained by the strategy formulated in Zugno et al. (2012).

In Figure 2, the resulting bids shown for different values of λ. As expected, the bids derived from the revenue formulation of the objective decreases with increased risk aversion. The fact that they don't go all the way down to 0 is a result of some of the scenarios containing negative down regulation price and that down regulation has a slightly higher probability than up regulation. However since the probability of the two penalising imbalance signs isn't severely different, the bid from the cost formulation doesn't change all that much with λ. The bid is nonetheless being pulled towards the point forecast but large tail of the up-regulation penalty density keeps it above the point forecast though for every value of λ. As shown in Figure 3, the CVaR behaves according to expectations from the bids. In Figure 4 the estimated density of the revenues from the 10000 scenarios is plotted for different values of lambda. As can be seen, the density doesn't change all that much on the scale of the plot. The expected value is decreasing though and the distribution goes from being two peaked to a single peak. However if some quantiles in the very leftmost tail of the distribution are viewed though some change becomes apparent as shown in Figure 5.

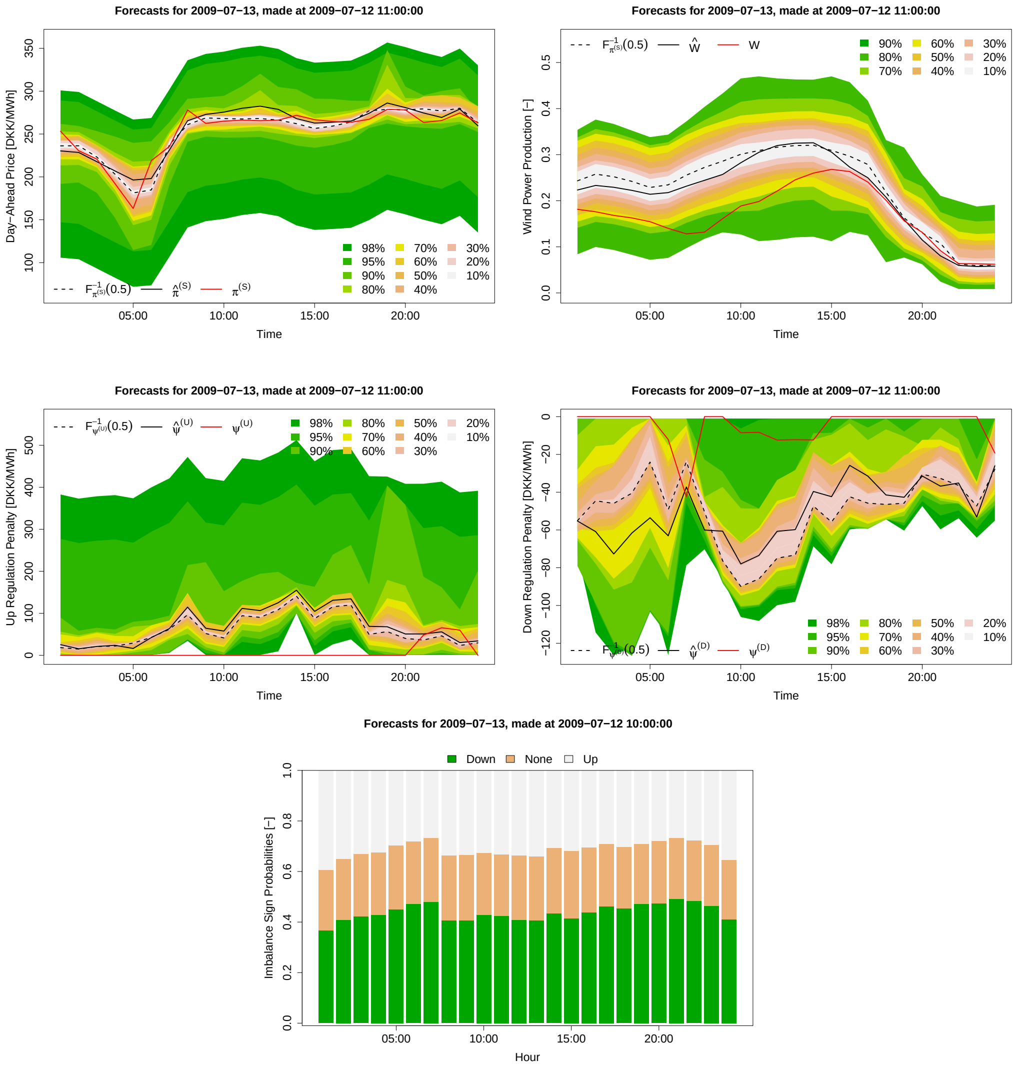

Figure 1: Forecasts for July 13th 2009 made at noon the day before

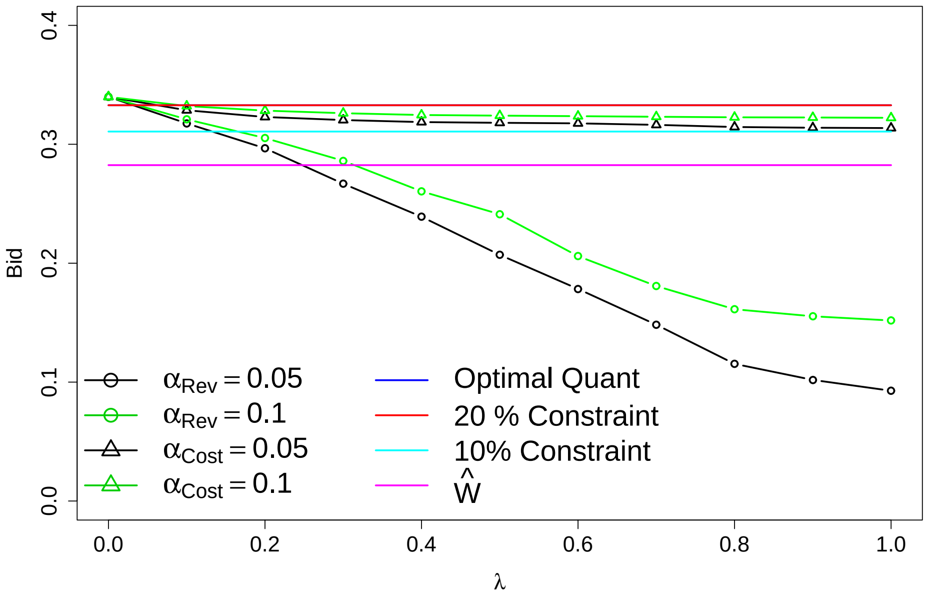

Figure 2: The optimal bids for the mean-CVaR bidding strategy for different values of λ along with bids from the newsvendor strategy. The unconditional optimal bid does not show since it is equal to the 20% constraint one.

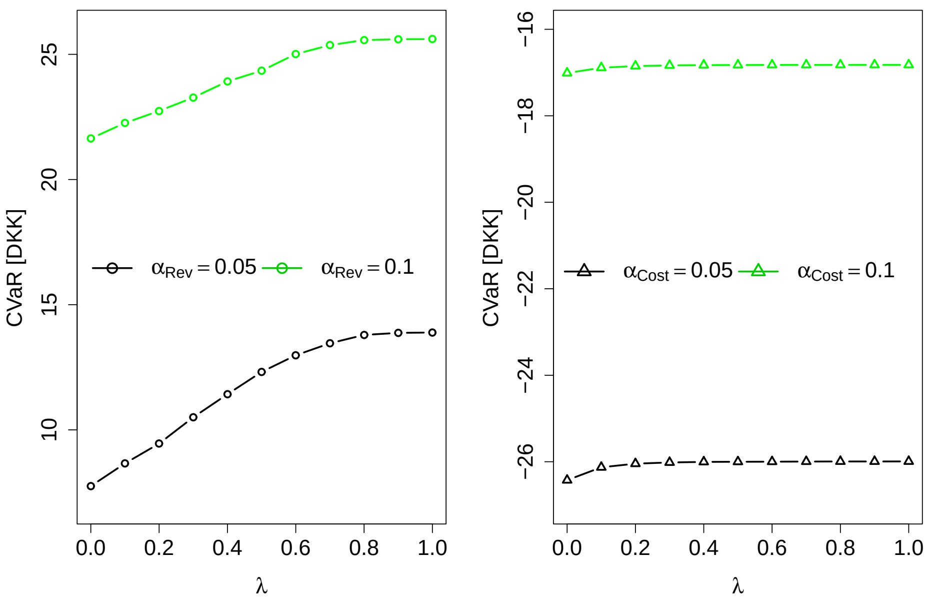

Figure 3: The CVaR for different values of λ and for the two formulations of the objective.

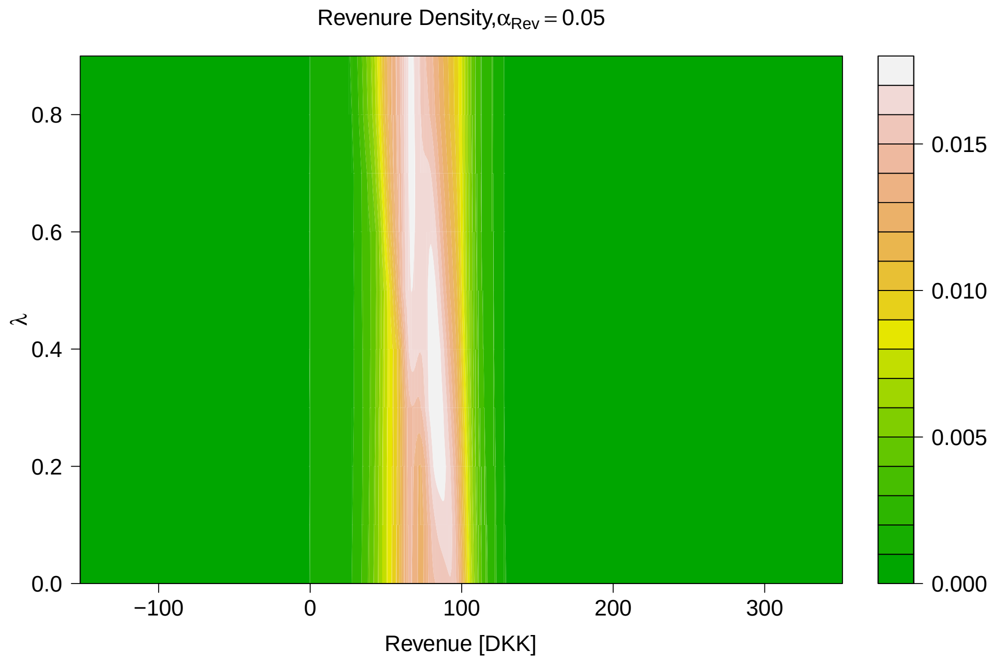

Figure 4: The density of the revenues for $\alpha = 0.05$ for different values of λ

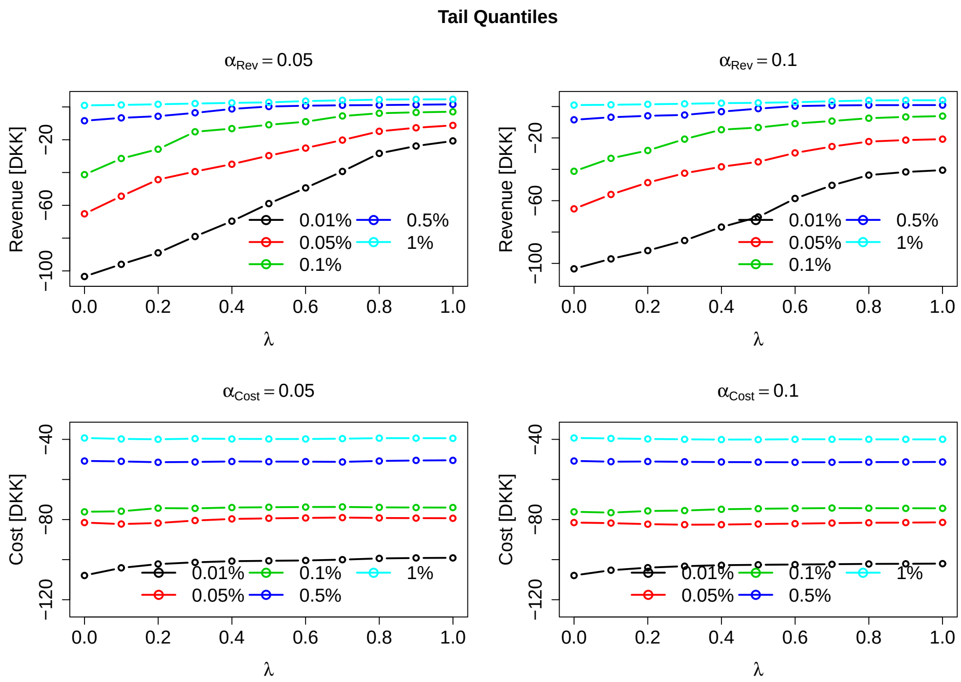

Figure 5: Movement in the leftmost tail of the cost/revenue distribution for different values of λ

7 Concluding Remarks

A framework for risk averse bidding of wind power to day-ahead electricity market has been derived. Should a risk-averse strategy for bidding be adopted, the CVaR seems like an appropriate choice due to it is concerned with the worst possible outcomes. This property is appealing for daily trading of wind power where the main focus of risk management should be on avoiding the few potential occurrences of excessive losses from the real-time market.

The framework here presented addresses the bidding and the risk on an hourly basis. Extending the framework to consider multiple periods simultaneously, e.g. 24 hours as done in Morales et al. (2010), is easy though. It simply involves swapping the hourly value for the sum of multiple ones within the terms of the objective functions. The scenario generation for such model would become considerably more complex since it would have to accommodate the temporal correlation of all the variables.

Regardless of whether the trading is considered on a daily or an hourly basis it is important to realise that long-term risk management goals are beyond reach with these types of strategies. This is also important to consider when the choice between the two formulations of the objective value presented is made. In light of all this, the relevancy and effect of short-term risk management in context with such efforts on longer time scales would be interesting to carry out.

References

John B. Bremnes. Probabilistic wind power forecasts using local quantile regression. *Wind Energy*, 7(1):47–54, 2004.

Youhua (Frank) Chen, Minghui Xu, and Zhe George Zhang. A Risk-Averse Newsvendor Model Under the CVaR Criterion. *Operations Research*, 57(4):1040–1044, 2009.

Chris J. Dent, Janusz W. Bialek, and Benjamin F. Hobbs. Opportunity cost bidding by wind generators in forward markets: Analytical results. *IEEE Transactions on Power Systems*, 26(3):1600–1608, 2011.

Robert F. Engle and Simone Manganelli. CAViaR: Conditional Autoregressive Value-at-Risk by Regression Quantiles. *Journal of Business and Economic Statistics*, 22:367–381, 2004.

W.L. Gorr and C. Hsu. An Adaptive Filtering Procedure for Estimating Regression Quantiles. *Management Science*, 31:1019–1029, 1985.

Tryggvi Jónsson, Pierre Pinson, Henrik Aalborg Nielsen, Henrik Madsen, and Torben Skov Nielsen. Forecasting electricity spot prices accounting for wind power predictions. *IEEE Transactions on Sustainable Energy*, (submitted), 2011.

Tryggvi Jónsson, Pierre Pinson, Henrik Aa. Nielsen, and Henrik Madsen. Exponential smoothing approaches for prediction in real-time electrcity markets. *Energy Economics*, (submitted), 2012a.

Tryggvi Jónsson, Pierre Pinson, Henrik Aa. Nielsen, and Henrik Madsen. Predictive densities for day-ahead electricity prices using time-adaptive quantile regression. *European Journal of Operational Research*, (submitted), 2012b.

Jan Kloppenborg Møller, Henrik Aalborg Nielsen, and Henrik Madsen. Time-adaptive quantile regression. *Computational Statistics and Data Analysis*, 51:1292–1303, 2008.

Juan M. Morales, Antonio J. Conejo, and Juan Pérez-Ruiz. Short-term trading for a wind power producer. *IEEE Transactions on Power Systems*, 25(1):554–564, 2010.

Holger Heitsch Nicole Growe-Kuska and Werner Romisch. Scenario reduction and scenario tree construction for power management problems. In *IEEE PowerTech Conference*, Bologna, Italy, June 23-26 2003.

Torben Skov Nielsen, Henrik Aalborg Nielsen, and Henrik Madsen. Prediction of wind power using time-varying coefficient functions. In *15th IFAC World Congress*, Barcelona, Spain, 2002.

Pierre Pinson, George Papaefthymiou, B. Klockl, Henrik Aalborg Nielsen, and Henrik Madsen. From probabilistic forecasts to statistical scenarios of short-term wind power production. *Wind Energy*, 12(1):51–62, 2009.

R. Tyrrell Rockafellar and Stanislav Uryasev. Optimization of Conditional Value-at-Risk. *The Journal of Risk*, 2(3):21–41, 2000.

R. Tyrrell Rockafellar and Stanislav Uryasev. Conditional Value-at-Risk for General Loss Distributions. *Journal of Banking and Finance*, 26(7):1443–1471, 2002.

Rudiger Schultz and Stephan Tiedemann. Conditional Value-at-Risk in Stochastic Programs with Mixed-Integer Recourse. *Mathematical Programming Series B*, 105:365–386, 2006.

James W. Taylor. Using Exponentially Weighted Quantile Regression to Estimate Value at Risk and Expected Shortfall. *Journal of Financial Econometrics*, 6:382–406, 2008.

Marco Zugno, Tryggvi Jónsson, and Pierre Pinson. Trading wind energy based on probabilistic forecasts of wind generation and market quantities. *Wind Energy*, (submitted), 2012.