Extreme winds in Denmark

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Extreme Winds in Denmark

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Abstract  Wind-speed data from four sites in Denmark have been analyzed in order to obtain estimates of the basic wind velocity which is defined as the 50-year wind speed under standard conditions, i.e. ten-minute averages at the height 10 m over a uniform terrain with the roughness length 0.05 m. The sites are, from west, Skjern (15 years), Kegnars (7 years), Sprogo (20 years), and Tystofte (15 years). The data are ten minute averages of wind speed, wind direction, temperature and pressure. The last two quantities are used to determine the air density $\rho$. The data are cleaned for terrain effects by means of a slightly modified WEP technique where the sector speed-up factors and roughness lengths are linearly smoothed with a direction resolution of one degree. Assuming geostrophic balance, all the wind-velocity data are transformed to friction velocity $u_*$ and direction at standard conditions by means of the geostrophic drag law for neutral stratification. The basic wind velocity in 30° sectors are obtained through ranking of the largest values of the friction velocity pressure $1/2\rho u_*^2$ taken both once every two months and once every year. The main conclusion is that the basic wind velocity is significantly larger at Skjern, close to the west coast of Jutland, than at any of the other sites. Irrespective of direction, the present standard estimates of 50-year wind are $25 \pm 1$ m/s at Skjern and $22 \pm 1$ m/s at the other three sites. These results are in agreement with those obtained by Jensen & Franck (1970) and Abild (1994) and supports the conclusion that the wind climate at the west coast of Jutland is more extreme than in any other part of the country. Simple procedures to translate in a particular direction sector the standard basic wind velocity to conditions with a different roughness length and height are presented. It is shown that a simple scheme makes it possible to calculate the total 50-year extreme load on a general structure without symmetry in a inhomogeneous terrain. A special section is devoted to the interpretation of the concepts in the Danish wind code DS 410 (1998) and Eurocode 1 (1995).
1 Introduction

The purpose of this investigation is to study, on basis of climatological records, how large extreme wind speeds are in various parts of Denmark. There is a suspicion that there are more strong winds at the west coast of Jutland than elsewhere in the country. We will try to see if this is actually the case by analyzing wind-speed records of contiguous ten-minute averages from four different places. Table 1 shows data pertinent to these sites.

Table 1. Position, height over local surface altitude and period.

<table>
<thead>
<tr>
<th>Position</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Height</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kegnås</td>
<td>54°51'20&quot;</td>
<td>09°56'11&quot;</td>
<td>23.4 m</td>
<td>19910101–19971231</td>
</tr>
<tr>
<td>Skjern</td>
<td>55°56'32&quot;</td>
<td>08°26'55&quot;</td>
<td>23.8 m</td>
<td>19820311–19971231</td>
</tr>
<tr>
<td>Sprogø</td>
<td>55°19'53&quot;</td>
<td>10°58'27&quot;</td>
<td>70.0 m</td>
<td>19770913–19971231</td>
</tr>
<tr>
<td>Tystofte</td>
<td>55°14'24&quot;</td>
<td>11°19'48&quot;</td>
<td>39.3 m</td>
<td>19820525–19971231</td>
</tr>
</tbody>
</table>

The positions are indicated in Fig. 1.

Figure 1. The four sites Kegnås, Skjern, Sprogø, and Tystofte (bullets). The Jensen-Franch sites Torsminde, Tune, and Gedser (open circles).

We define the extreme wind speeds in terms of the 50-year wind speed, i.e. the wind speed which, on average, is exceeded once in a period of 50 years. This quantity is determined, by ranking of maximum values from records of a particular duration, for 12, 30° wind-direction sectors and for all directions.

In order to compare the extreme winds from the four sites we must ‘transform’ the measured data to a common reference height and a common surface roughness length $z_0$. We use the 50-year, ten-minute mean wind speed at the height 10 m over a uniform terrain with $z_0 = 0.05$ m as the reference and call it the basic wind velocity. The transformation to this reference is meaningful if we assume geostrophic balance so that the driving force is the wind speed outside the boundary layer, the so-called geostrophic wind $G$. The transformed data in the form of the height-independent friction velocity $u_*$ is obtained by applying the WAEP
method (Mortensen et al. 1993) for ‘correcting’ the measured data for orographic terrain effects and roughness changes and then ‘mapping’ the friction velocity to a standard surface roughness length by means the geostrophic drag law for neutral stratification.

The idea of using WAP techniques to transform measured data to ‘standard’ data with the purpose of determining ‘standard’ extreme wind speeds is by no means new. Abild & Nielsen (1991), Abild et al. (1992), and Abild (1994) applied and discussed in depth this method. These authors obtained their extreme wind speeds by means of ranking of the Annual Maxima (the AM method) and by means of the Peak-Over-Threshold method (POT). Abild (1994) in particular also discusses the underlying statistical basis for these methods and later Mann et al. (1998) provide the same information in an abbreviated form.

We have not carried the extreme value analysis out on the wind speed, but rather on the friction velocity pressure

\[ q = \frac{1}{2} \rho u^2_z, \]

where \( \rho \) is the air density. We use the simultaneous records of temperature \( T \) and pressure \( p \) to determine this quantity for each wind velocity measurement.

In the following we briefly discuss the methodology of the transformation of the data and the ranking method of determining the extreme wind speeds. We then present the results and compare the 50-year winds from the four sites.
2 Data Transformation

As mentioned in the introduction, the transformation to a standard friction velocity consists of two steps: ‘WASP cleaning’ and ‘geostrophic mapping’.

2.1 WASP Cleaning

The purpose of this cleaning is to determine from the observed wind speed $U_0(z)$ the so-called free-stream wind speed $U(z)$. This is defined as the wind speed which would be observed at the measuring height $z$ if there were no obstacles and no roughness changes and if the terrain were flat. The standard WASP method prescribes in each of the 12 direction sectors an average roughness length $z_0$ and the relative speed-up increases $\Delta u_\phi/U$ and $\Delta u_r/U$, due to orographic effects and roughness changes, respectively.

Together with $U_0(z)$ we observe the wind direction $D_0$ and we want to assign to this observed direction speed-up increases and a roughness. We could use those pertaining to the sector corresponding to $D_0$, but this might create rather large jumps in the free-stream wind speed $U(z)$ and the corresponding friction velocity $u_{r0}$ when the direction changes from one sector to the neighboring sector. We therefore smooth out $z_0$ and $S_\phi = \Delta u_\phi/U$ and $S_r = \Delta u_r/U$ and give them with the same resolution as the measured direction $D_0$, namely $1^\circ$. The relation between an unsmoothed quantity $f(D_0)$ and the corresponding smoothed quantity $F(D_0)$ is

$$F(D_0) = \frac{1}{\Delta} \int_{D_0-\Delta/2}^{D_0+\Delta/2} f(D_0') \, dD_0', \quad (2)$$

where $\Delta = 30^\circ$ is the magnitude of the direction sector.

Figure 2 shows an example of the smoothing, in this case the roughness length $z_0(D_0)$ at Tystofte. We see that the smoothed roughness length is a continuous function of direction.

The WASP speed-ups and roughness lengths for the four stations are given in Table 2.

Table 2. The speed-ups and roughness lengths at the four sites.

| $D_0$ | $S_\phi, \%$ | $S_r, \%$ | $z_00$, m | $S_\phi, \%$ | $S_r, \%$ | $z_00$, m | $S_\phi, \%$ | $S_r, \%$ | $z_00$, m | $S_\phi, \%$ | $S_r, \%$ | $z_00$, m |
|-------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 00°   | +1.380      | -2.740      | 0.044136    | +0.872      | +2.315      | 0.117185    | +0.690      | 0.000       | 0.000205    | -0.704      | +1.194      | 0.115490    |
| 30°   | +1.011      | -2.231      | 0.012196    | +0.922      | +0.000      | 0.005388    | +0.689      | +0.000      | 0.000207    | -0.964      | +0.671      | 0.112942    |
| 60°   | +1.868      | -12.454     | 0.001438    | +0.194      | +0.000      | 0.109245    | -0.718      | +1.078      | 0.000597    | -0.316      | +0.000      | 0.305315    |
| 90°   | +1.927      | -12.703     | 0.000812    | +0.298      | -2.015      | 0.005063    | -0.937      | +1.165      | 0.000659    | +0.567      | +0.000      | 0.841455    |
| 120°  | +1.529      | 0.000       | 0.000216    | -0.145      | +2.403      | 0.008240    | -0.935      | 0.000       | 0.000200    | +0.752      | -0.818      | 0.009829    |
| 150°  | +1.628      | 0.000       | 0.000205    | -0.722      | +3.963      | 0.001012    | +0.440      | 0.000       | 0.000200    | +0.138      | -0.991      | 0.001962    |
| 180°  | +1.950      | 0.000       | 0.000203    | -0.861      | +4.192      | 0.005581    | +0.659      | 0.000       | 0.000200    | -0.633      | +0.165      | 0.001340    |
| 210°  | +2.129      | 0.000       | 0.000203    | -0.427      | +6.200      | 0.014243    | +0.988      | +0.379      | 0.000200    | -0.995      | +1.309      | 0.002770    |
| 240°  | +1.990      | 0.000       | 0.000204    | +0.158      | +4.670      | 0.001828    | -0.726      | +4.236      | 0.001916    | -0.315      | +1.069      | 0.000681    |
| 270°  | +1.686      | 0.000       | 0.000208    | +0.246      | -6.501      | 0.001276    | -0.998      | +4.301      | 0.004334    | +0.493      | +8.371      | 0.005107    |
| 300°  | +1.737      | -2.600      | 0.000692    | -0.135      | -8.696      | 0.005353    | -0.356      | +0.441      | 0.009298    | +0.784      | +13.530      | 0.007793    |
| 330°  | +1.512      | -5.948      | 0.007831    | -0.731      | -9.910      | 0.008049    | +0.441      | 0.000       | 0.000200    | +0.198      | -3.163      | 0.087416    |

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We obtain the free-stream wind speed by the equation
\[ U(z) = \frac{U_0(z)}{(1 + S_a(D_0))(1 + S_r(D_0))} \quad (3) \]
and, assuming the well-known logarithmic wind-profile for neutral stratification, the friction velocity is determined by
\[ u_{*0}(D_0) = \frac{\kappa U(z)}{\ln(z/z_{00}(D_0))} \quad (4) \]
where \( \kappa \) is the von Kármán constant, here assumed equal to 0.4.

2.2 Geostrophic Mapping

In order to determine the magnitude \( G \) and direction of the geostrophic wind we use the approach described by Troen & Petersen (1989). Tennekes (1982) has given an elegant derivation of the equations for the geostrophic drag law.

\[ G = \frac{u_{*0}}{\kappa} \sqrt{\left( \ln \left( \frac{u_{*0}}{f z_{00}} \right) - A \right)^2 + B^2} \quad (5) \]

where \( f = 2 \times (\text{the earth's rate of rotation in radians per second}) \times \sin(\text{latitude}) \approx 1.2 \times 10^{-4} \text{ rad/s at latitude } 55.5^\circ \), and where \( A \) and \( B \) are dimensionless constants. The numerical values \( A = 1.8 \) and \( B = 4.5 \) are recommended by Mortensen et al. (1993) and Troen & Petersen (1989) and therefore used in the present analysis.

The geostrophic wind will in general have another direction than the surface wind. At the northern hemisphere the wind direction will turn clockwise from the surface wind up through the boundary layer to the geostrophic wind. The angle \( \Delta D_0 \) from

\[ z_{00}(D_0) \]
\[ 0.00 \quad 0.05 \quad 0.10 \quad 0.15 \quad D_0 \]
\[ 000^\circ \quad 030^\circ \quad 060^\circ \quad 090^\circ \quad 120^\circ \quad 150^\circ \quad 180^\circ \quad 210^\circ \quad 240^\circ \quad 270^\circ \quad 300^\circ \quad 330^\circ \quad 360^\circ \]

Figure 2. Smoothed (thick line) and unsmoothed (thin line) roughness lengths \( z_{00} \) as function of the direction \( D_0 \) at Tystofte.
the direction of surface wind to that of the geostrophic wind is given by

\[
\begin{align*}
\begin{bmatrix}
\cos(\Delta D_0) \\
\sin(\Delta D_0)
\end{bmatrix} &= 
\frac{1}{\sqrt{\left(\ln\left(\frac{u_*}{fz_0}\right) - A\right)^2 + B^2}} \times \begin{bmatrix}
\ln\left(\frac{u_*}{fz_0}\right) - A
\end{bmatrix}.
\end{align*}
\] (6)

Geostrophic balance implies that the geostrophic wind velocity is terrain independent and it is consequently possible to determine the friction velocity \(u_*\) over any roughness lengths \(z_0\) by solving

\[
G = \frac{u_*}{\kappa} \sqrt{\left(\ln\left(\frac{u_*}{fz_0}\right) - A\right)^2 + B^2}.
\] (7)

Actually, we solve, by means of the Newton method, the equation

\[
x^2 \left\{ (\ln(x) + \ln(\kappa Ro) - A)^2 + B^2 \right\} - 1 = 0.
\] (8)

with the surface Rossby number

\[
Ro = \frac{G}{fz_0}
\] (9)
as parameter, for

\[
x = \frac{u_*}{\kappa G}
\] (10)

Once we have determined \(u_*\), we can calculate the angle \(\Delta D\) from the surface wind to the geostrophic wind by means of

\[
\begin{align*}
\begin{bmatrix}
\cos(\Delta D) \\
\sin(\Delta D)
\end{bmatrix} &= 
\frac{1}{\sqrt{\left(\ln\left(\frac{u_*}{fz_0}\right) - A\right)^2 + B^2}} \times \begin{bmatrix}
\ln\left(\frac{u_*}{fz_0}\right) - A
\end{bmatrix}.
\end{align*}
\] (11)

The direction \(D\) of the surface wind over the roughness length \(z_0\) becomes

\[
D = D_0 + \Delta D_0 - \Delta D.
\] (12)

Here we will use the the value \(z_0 = 0.05\) m for all directions.
3 Data

As Table 1 shows, the longest time record is from Sprogø (about 20 years). Here the data consists of wind speed, wind direction, temperature and pressure. As mentioned in the introduction, the last two quantities are used to calculate the air density record which is necessary for the determination of the wind velocity pressure $q$ in (1).

The data records from Skjern and Tystofte have about equal durations (about 16 years), but pressure is not measured at Tystofte.

The shortest data record is from Kegnæs (7 years).

At a particular station at a particular time, a missing temperature or a missing pressure measurement is supplied by the corresponding average from the other stations.

Data can be missing for a number reasons: power failure, sensor errors, servicing of the installation etc. Table 3 shows how many data points are actually retrieved in the sense that both direction and speed are measured.

Table 3. Percentage data covering.

<table>
<thead>
<tr>
<th>Year</th>
<th>Kegnæs</th>
<th>Skjern</th>
<th>Sprogø</th>
<th>Tystofte</th>
</tr>
</thead>
<tbody>
<tr>
<td>1977</td>
<td>99.90</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1978</td>
<td>99.90</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1979</td>
<td>93.54</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1980</td>
<td>84.41</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1981</td>
<td>89.05</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1982</td>
<td>90.26</td>
<td>99.84</td>
<td>94.00</td>
<td></td>
</tr>
<tr>
<td>1983</td>
<td>97.22</td>
<td>99.93</td>
<td>90.39</td>
<td></td>
</tr>
<tr>
<td>1984</td>
<td>99.86</td>
<td>99.92</td>
<td>99.68</td>
<td></td>
</tr>
<tr>
<td>1985</td>
<td>99.95</td>
<td>99.94</td>
<td>94.21</td>
<td></td>
</tr>
<tr>
<td>1986</td>
<td>88.38</td>
<td>99.94</td>
<td>99.97</td>
<td></td>
</tr>
<tr>
<td>1987</td>
<td>87.47</td>
<td>99.96</td>
<td>99.94</td>
<td></td>
</tr>
<tr>
<td>1988</td>
<td>95.06</td>
<td>92.13</td>
<td>97.25</td>
<td></td>
</tr>
<tr>
<td>1989</td>
<td>91.29</td>
<td>96.62</td>
<td>91.97</td>
<td></td>
</tr>
<tr>
<td>1990</td>
<td>99.98</td>
<td>99.98</td>
<td>92.41</td>
<td></td>
</tr>
<tr>
<td>1992</td>
<td>99.95</td>
<td>99.67</td>
<td>99.39</td>
<td>95.39</td>
</tr>
<tr>
<td>1993</td>
<td>98.35</td>
<td>95.37</td>
<td>99.90</td>
<td>99.89</td>
</tr>
<tr>
<td>1994</td>
<td>95.34</td>
<td>99.87</td>
<td>99.67</td>
<td>99.98</td>
</tr>
<tr>
<td>1995</td>
<td>99.99</td>
<td>98.83</td>
<td>99.97</td>
<td>99.63</td>
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<tr>
<td>1996</td>
<td>99.98</td>
<td>99.89</td>
<td>99.97</td>
<td>97.64</td>
</tr>
<tr>
<td>1997</td>
<td>99.99</td>
<td>99.98</td>
<td>99.80</td>
<td>96.90</td>
</tr>
</tbody>
</table>

Nothing has been done to replace missing wind data.
4 Analysis

Here we use the ranking procedure which is defined in the following way:

1. For each site we subdivide the entire record in $M$ smaller sub-records of identical durations $T_0$.
2. We select in each sub-record the largest value $q_n$ of $q$ in each $30^\circ$ direction sector, centered around $n \times 30^\circ$ with $n = 0, 1, \ldots, 11$, and the largest value $q_{12}$ irrespective of direction. This amounts to 13 sets of largest values.
3. For each set we rank the maximum values $q_n[m]$, $m = 1, 2, \ldots, M$, and plot these versus $-\ln(-\ln(m/(M + 1)))$.
4. We fit a straight line through each plot and determine the zero offset $\tilde{q}_n(T_0)$ and the slope $\alpha_n$ ($n = 0, 2, \ldots, 12$).
5. We determine the value $\tilde{q}_n(T)$ which on average is exceeded once in the period $T \neq T_0$.

With reference to item 3 we follow the recommendation by Gumbel (1958). This is concerned with the probability assignment to $q_n[m]$. Here we shall try to shed light on the arguments leading to this assignment.

If the probability density and the cumulative probability for the maximum value $-\infty < q < \infty$ in a record of duration $T_0$ are $p(T_0, q)$ and

$$P(T_0, q) \equiv \int_{-\infty}^{q} p(T_0, q') \, dq',$$  \hfill (13)

respectively, then the probability density for the $m$th ranked value becomes

$$\varphi(q; M, m) = M \left( \frac{M - 1}{m - 1} \right) P^{m-1}(T_0, q) \{ 1 - P(T_0, q) \}^{M-m} p(T_0, q).$$  \hfill (14)

We imagine that we have an infinite ensemble of $M$ ranked variables $q[m]$. Then we can define an ensemble average $\langle q[m] \rangle$ of $q[m]$. With the probability density (14) this average becomes

$$\langle q[m] \rangle = \int_{-\infty}^{\infty} q \varphi(q; M, m) \, dq.$$  \hfill (15)

We consider the actual value $q[m]$ from the one trial we have as the best approximation to $\langle q[m] \rangle$.

Gumbel (1958) argues that the probability that the $m$th value is less than $\langle q[m] \rangle$ should be $P(T_0, \langle q[m] \rangle)$ rather than $P(T_0, q[m])$, mostly because it is independent of the actual form of $P(T_0, q)$ and because it is easy to calculate.

We get
\[ 
\langle P(T_0, q[m]) \rangle = \int_{-\infty}^{\infty} P(T_0, q) \varphi(q; M, m) \, dq 
\]

\[ 
= \int_{-\infty}^{\infty} P(T_0, q) \left( \frac{m}{M} \right) m P^{m-1}(T_0, q) \{ 1 - P(T_0, q) \}^{M-m} p(T_0, q) \, dq 
\]

\[ 
= \left( \frac{M}{m} \right) m \int_{0}^{1} P^{m} \{ 1 - P \}^{M-m} \, dP 
\]

\[ 
= \left( \frac{M}{m} \right) m \frac{m! (M - m)!}{(M + 1)!} = \frac{m}{M + 1} \quad (16) 
\]

We assume that the extreme events have an accumulated probability which is a double exponential (Gumbel 1958), i.e.

\[ 
P(T_0, q) = \exp(\exp(-q/\tilde{q}/\alpha)), \quad (17) \]

so if this is really the case then \( q[m] \) plotted versus \( -\ln(-\ln(m/(M + 1))) \) should, on average, be lying on a straight line with the offset \( \tilde{q} \) and the slope \( \alpha \). This is the background for the statement in item 4.

The quantity \( \tilde{q} \) is the most probable value of \( q[m] \) (the mode of \( p(T_0, q) \)) and also the value of \( q \) which, on average, is exceeded once in the period \( T_0 \) [see e.g., Kristensen et al. (1991)]. We see that the probability assignment is practical also because is fulfills the condition that also the largest value can be plotted. This would not be the case if we had assigned the value \( m/M \) to the accumulated probabilities.

The argument of the outer exponential function in (17) can also, under the assumption that the individual excursions of \( q \) are statistical independent, be interpreted as minus the average number of times \( \mathcal{N}(T_0) \) the quantity exceeds the value \( q \) in a period of time \( T_0 \). In other words

\[ 
\mathcal{N}(T_0) = -\ln(P(T_0, q)) = e^{-\exp(-q/\tilde{q}/\alpha)} \quad (18) 
\]

Since this number for a stationary time series is proportional to the duration, we can determine \( \mathcal{N}(T) \) for any time \( T \) by using the identity

\[ 
\frac{\mathcal{N}(T)}{T} = \frac{\mathcal{N}(T_0)}{T_0} \quad (19) 
\]

Inserting the exponential (18) we get

\[ 
\frac{e^{-\exp(\tilde{q}/\alpha)}}{T_0} = \frac{e^{-\exp(\tilde{q}/\alpha)}}{T} \quad (20) 
\]

so that

\[ 
\tilde{q}(T) = \tilde{q}(T_0) + \alpha \ln \left( \frac{T}{T_0} \right) \quad (21) 
\]

This is the equation we use to determine the extreme value \( \tilde{q} \) when the duration of the record is different from \( T_0 \).
As (14) shows, the probability density for the ranked variables \( q[m] \) depends on \( m \) as well as the total number of observations: when \( m \) is close to \( M \) or 1, \( \varphi(q; M, m) \) is wider than when \( m \) is close to \( M/2 \) because in the last case there is 'less room to move around than at the end points'. In the fitting, the points should have different weights. It is convenient in our case to use fractiles of \( \varphi(q; M, m) \) to calculate the weights \( w[M, m] \). We have chosen the 68% fractiles, corresponding to the 'probability mass' within \( \pm \) the standard deviation of the normal probability density function. This means that we use the values, \( q_+[M, m] \) and \( q_-[M, m] \), of \( q \) for which we have

\[
\int_{-\infty}^{q_+[M, m]} \varphi(q; M, m) \, dq = \frac{1}{2} \left[ 1 + \text{erf}\left( \frac{1}{\sqrt{2}} \right) \right] = G_+ \tag{22}
\]

and

\[
\int_{-\infty}^{q_-[M, m]} \varphi(q; M, m) \, dq = \frac{1}{2} \left[ 1 - \text{erf}\left( \frac{1}{\sqrt{2}} \right) \right] = G_- \tag{23}
\]

where

\[
\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-s^2} \, ds \tag{24}
\]

is the error function (Gautschi 1964).

First we use (14) to obtain for any of the variables \( q = q_+[M, m], q_-[M, m] \), corresponding to \( G = G_+, G_- \),

\[
G = \int_{-\infty}^{q} \varphi(q'; M, m) \, dq' = \frac{B_P(T_0, q)(m, M - m + 1)}{B(m, M - m + 1)}, \tag{25}
\]

where \( B_P(a, b) \) and \( B(a, b) = B_1(a, b) \) are the incomplete and the complete beta function (Davis 1964) where \( P(T_0, q) \) is given by (17).

Then we solve this equation numerically by means of a generalized Newton method (Brent 1969) for \( P(T_0, q) \). The two solutions corresponding to \( G_+ \) and \( G_- \) are \( P_+[M, m] \) and \( P_-[M, m] \), respectively.

Inverting (17), we get

\[
\frac{q}{\alpha} = \frac{\tilde{q}}{\alpha} - \ln(-\ln(P(T_0, q))) \tag{26}
\]

The 'width' of \( \varphi(q; M, m) \), within which we have 68% of the probability mass, is

\[
\Delta q[M, n] = q_+[M, m] - q_-[M, m] = \alpha \ln \left( \frac{\ln(P_-[M, m])}{\ln(P_+[M, m])} \right) \tag{27}
\]

A priori we don't know \( \alpha \), but since only the relative weights are necessary we use

\[
w[M, m] = \left( \frac{\alpha}{\Delta q[M, m]} \right)^2 \tag{28}
\]

The quantity \( \Delta q[M, m]/\alpha \), proportional to the width of \( \varphi(q; M, m) \), is shown in Fig. 3 for \( M = 20 \).
Once $\alpha$ has been determined we can calculate the true width of $\varphi(q; M, m)$ which is approximately equal to twice the standard deviation. This, in turn, enables us to calculate the standard deviations $\sigma(\alpha)$ and $\sigma(\varphi(T_0))$ of $\alpha$ and $\varphi(T_0)$ and the correlation $\mu(T_0)$ between these two quantities (Arley & Buch 1969). We will then be able to find the standard deviation of any function $g(\alpha, \varphi(T_0))$ of $\alpha$ and $\varphi(T_0)$:

\[
\sigma(g(\alpha, \varphi(T_0))) = \sqrt{g'^2 \sigma^2(\alpha) + 2g'g'' \mu(T_0) \sigma(\alpha) \sigma(\varphi(T_0)) + g''^2 \sigma^2(\varphi(T_0))},
\]

(29)

where

\[
\begin{align*}
\left\{ g'_1 \right\} &= \left\{ \frac{\partial g}{\partial \alpha} \right\} g(\alpha, \varphi(T_0)), \\
\left\{ g'_2 \right\} &= \left\{ \frac{\partial g}{\partial \varphi(T_0)} \right\} g(\alpha, \varphi(T_0)).
\end{align*}
\]

(30)

Applying (29) to (21) we get

\[
\sigma(\varphi(T)) = \sqrt{\sigma^2(\varphi(T_0)) + 2 \ln \left( \frac{T}{T_0} \right) \mu(T_0) \sigma(\varphi(T_0)) \sigma(\alpha) + \left\{ \ln \left( \frac{T}{T_0} \right) \right\}^2 \sigma^2(\alpha)}.
\]

(31)
5 Results

The records from all the sites (Table 1), except the rather short record from Kegness, have been subdivided in two different ways: \( T_0 = 61 \) days and \( T_0 = 1 \) year. We have chosen the first because it provides large ensembles and because it is convenient that the time ratio between the first and the last is very close to six. The second period is chosen in order to check the results for effects of seasonal variations. For each of the sites we follow the procedure outlined in section 4. The detailed results, figures and tables, for each site are included in the appendix.

The extreme statistics is carried out on the friction velocity pressure (1) assuming a roughness length \( z_0 \) uniformly equal to \( z_0 = 0.05 \) m. We translate the 50-year friction velocity pressure \( q(T) \) \( (T = 50 \) years) to the equivalent basic wind velocity \( U_{50} \) at the height \( z = 10 \) m with the standard air density \( \rho_0 = 1.25 \) kg/m\(^3\) by the equation

\[
U_{50} = \frac{1}{\kappa} \sqrt{\frac{2g}{\rho_0}} \ln \left( \frac{z}{z_0} \right),
\]

(32)

where the von Kármán constant is assumed equal to 0.4.

5.1 Sprogø

The record from Sprogø is the longest; there are \( M = 121 \) 61-day periods and \( M = 20 \) one-year periods. Figure 10 shows the plot of the 121 ordered values of \( q \) in each of the 12 direction sectors.

Figure 12 shows the basic wind velocity, based on 61-day periods, in the 12 direction sectors.

When \( T_0 = 1 \) year the corresponding plots are shown in Figs. 13 and 15.

The results from Sprogø are summarized in Table 7.

Table 7 shows good agreement, for all direction sectors and also for all directions together \((000^\circ - 360^\circ)\), between the determination of the 50-year wind determined on basis of \( T_0 = 61 \) days and \( T_0 = 1 \) year. Since the number of realizations are about six times larger in the first case, the 68% confidence limits are correspondingly smaller.

5.2 Skjern

The record from Skjern has \( M = 94 \) 61-day periods and \( M = 15 \) one-year periods. Figure 16 shows the plot of the 94 ordered values of \( q \) in each of the 12 direction sectors.

Figure 18 shows the basic wind velocity, based on 61-day periods, in the 12 direction sectors.

We see immediately in Fig. 16 that there are severe discrepancies between the assumption about the double exponential probability and the data from the entire western semicircle. There seems to be two distinct domains. For large values of \( -\ln(-\ln(m/(M+1))) \) the extreme values of \( q \) is much larger than predicted by the lower values of \( -\ln(-\ln(m/(M+1))) \). A closer inspection of the extreme values
shows that all the large values of \( q \) in the western semicircle occur during the first and the last 61-day periods of the calendar year. We must therefore conclude that the 61-day periods are not taken from the same population, possibly because the seasonal variation of the surface roughness has a significant impact at Skjern where the measuring height is rather low. We saw that at Sprogø the sample durations \( T_0 = 61 \) days and \( T_0 = 1 \) year gave consistent 50-year winds. This is obviously not the case here and another independent estimate seems to be called for. Consequently we have applied the POT method to the measured velocity pressure data at Skjern and found a basic wind velocity of about 25 to 26 m/s. This estimate is consistent with the basic wind velocity determined on basis of \( T_0 \) equal to 61 days. Inspection of Fig. 20 supports this point of view in that the lower values of the extremes have too much influence on the slope of the line.

The result of the analysis for \( T_0 = 61 \) days is shown in Figs. 16 and 18.

A wind tunnel investigation was carried out to see if the shelter belts, consisting of 4 to 7 m high trees and bushes, at Skjern would cause a wind speed-up at the measuring height. The result was that the shelter belts could have only very little influence on the measurements.

5.3 Tystofte

The record from Tystofte has \( M \approx 93 \) 61-day periods and \( M \approx 15 \) one-year periods. Figures 22, 24, 25, and 27 are plots showing the results of the analysis.

Table 11 shows that the resulting basic wind velocity at Tystofte for \( T_0 = 61 \) days and \( T_0 = 1 \) are mutually consistent, just as in the case of Sprogø.

5.4 Kegnæs

The record from Kegnæs has \( M \approx 42 \) 61-day periods. Figure 28 shows the plot of the 42 ordered values of \( q \) in each of the 12 direction sectors.

5.5 Summary of Analysis

We can now summarize the result of the analysis by presenting the 50-year winds for the four sites.

Table 4 shows that there is a general agreement between the basic wind velocity at Sprogø, Tystofte and Kegnæs for all direction sectors and all directions together (last row). However, at Skjern the basic wind velocity is significantly larger in the western semi-circle and for all directions together.

As pointed out the extreme wind speeds at Skjern are occurring in the four month around winter solstice. One explanation could be that in this period the upstream roughness length \( z_0 \) towards west, for some reason (flooding), is smaller than anticipated in these calculations.

However, most likely we must accept the folklore that the wind climate is tougher at the west coast of Denmark than elsewhere in the country.

We have estimated the seasonal variation of the basic wind velocity by carrying out a more detailed analysis of the data record at Sprogø because here we have the longest record. We used again temporal sections of 61-days duration. The basic
wind velocity now depends not only on the wind direction but also on which of the six 61-days period is being considered. Roughly speaking, we may call these six periods, numbered from 0 to 5, January–February, March–April, May–June, July–August, September–October, and November–December.

The result of this analysis is given in Table 5 and Fig. 4.

**Table 5. The basic wind velocity as a function of direction (rows) and period (columns). The last row corresponds to all directions and the last rightmost column to all year.**

<table>
<thead>
<tr>
<th>Period</th>
<th>00</th>
<th>01</th>
<th>02</th>
<th>03</th>
<th>04</th>
<th>05</th>
<th>00-05</th>
</tr>
</thead>
<tbody>
<tr>
<td>00°</td>
<td>15 ± 1</td>
<td>18 ± 1</td>
<td>16 ± 1</td>
<td>16 ± 1</td>
<td>16 ± 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>030°</td>
<td>14 ± 1</td>
<td>16 ± 1</td>
<td>14 ± 1</td>
<td>17 ± 1</td>
<td>17 ± 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>060°</td>
<td>15 ± 1</td>
<td>19 ± 1</td>
<td>17 ± 1</td>
<td>17 ± 1</td>
<td>17 ± 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>090°</td>
<td>17 ± 1</td>
<td>19 ± 1</td>
<td>17 ± 1</td>
<td>18 ± 1</td>
<td>18 ± 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>120°</td>
<td>16 ± 1</td>
<td>16 ± 1</td>
<td>15 ± 1</td>
<td>17 ± 1</td>
<td>17 ± 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>150°</td>
<td>17 ± 1</td>
<td>18 ± 1</td>
<td>15 ± 1</td>
<td>19 ± 1</td>
<td>19 ± 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>180°</td>
<td>18 ± 1</td>
<td>19 ± 1</td>
<td>18 ± 1</td>
<td>20 ± 1</td>
<td>20 ± 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>210°</td>
<td>19 ± 1</td>
<td>20 ± 1</td>
<td>20 ± 1</td>
<td>21 ± 1</td>
<td>21 ± 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>240°</td>
<td>19 ± 1</td>
<td>21 ± 1</td>
<td>20 ± 1</td>
<td>21 ± 1</td>
<td>21 ± 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>270°</td>
<td>20 ± 1</td>
<td>21 ± 1</td>
<td>21 ± 1</td>
<td>22 ± 1</td>
<td>22 ± 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>300°</td>
<td>19 ± 1</td>
<td>21 ± 1</td>
<td>16 ± 1</td>
<td>24 ± 1</td>
<td>24 ± 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>330°</td>
<td>16 ± 1</td>
<td>19 ± 1</td>
<td>16 ± 1</td>
<td>20 ± 1</td>
<td>20 ± 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>000°–360°</td>
<td>21 ± 1</td>
<td>23 ± 1</td>
<td>22 ± 1</td>
<td>25 ± 1</td>
<td>25 ± 1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 4 shows that at Sprogö the seasonal modulation of the basic wind velocity may be approximately described by the function

$$M(t) = 1 + \frac{1}{6} \times \cos \left( 2\pi \frac{t}{12} \right),$$

where $t$ is the time measured in months, starting from the beginning of the year.
Figure 4. Seasonal variation of the basic wind velocity. The thin lines correspond to particular direction sectors and the thick line to all directions.

We see that at midsummer where the standard 50-year wind speed reaches its minimum the reduction is about 16% compared to the mean.
6 Other Investigations

There are two independent investigations concerned with the extreme-wind conditions in Denmark, that by Jensen & Franck (1970) and the aforementioned, mainly by Abild (1994). Here we compare our results with the outcome of these investigations.

6.1 Jensen and Franck

In the period from the beginning of 1959 until the end of 1967, Jensen & Franck (1970) operated three measuring stations with the purpose of studying the climate of extreme winds in Denmark (see Fig. 1). Their data have been used as basis for basic wind velocity specified in the old Danish wind code DS 410 (1982). They used a so-called Dines anemometer (Middleton 1969), mounted at the top of 25 m masts at Torsminde at the west coast of Jutland, at Gedser, and at Tønne near Roskilde. By a sophisticated, simple design they were able to measure the daily maximum velocity pressure and nothing else.

According to Jensen & Franck (1970), the instrumentation time constant gave velocity-pressure averages over 3 to 5 seconds. Using ESDU 83045 (1983) (Engineering Sciences Data Unit), the ratio between a 3 to 5 seconds wind velocity $U_{3-5s}$ and the ten-minutes mean velocity $U_{10\text{ min}}$ is given by

$$\frac{U_{3-5s}}{U_{10\text{ min}}} = (1 + k_p I_u) \times 0.945,$$

where $k_p$ is a peak factor of approximately 2.9 and $I_u$ is the turbulence intensity given in the table below.

The 50-year velocity pressures $q_{3-5s}$ estimated at the three stations, are given below. The basic wind velocities in the rightmost column are calculated using the velocity pressure measured and the conversion method (34) indicated above together with the methods discussed in subsection 7.1.

Table 6. Summary of the results of Jensen and Franck and their interpretation.

<table>
<thead>
<tr>
<th>Station</th>
<th>$z_0$ (m)</th>
<th>$I_u$ (ESDU)</th>
<th>$q_{3-5s}$ (Pa)</th>
<th>$U_{50}$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tønne</td>
<td>0.05</td>
<td>0.17</td>
<td>873</td>
<td>22.3</td>
</tr>
<tr>
<td>Gedser</td>
<td>0.001-0.01</td>
<td>0.11-0.14</td>
<td>1030</td>
<td>22.7</td>
</tr>
<tr>
<td>Torsminde</td>
<td>0.001-0.01</td>
<td>0.11-0.14</td>
<td>1256</td>
<td>25.1</td>
</tr>
</tbody>
</table>

These values of $U_{50}$ are within one standard deviation the same as found in the present analysis of data from Kegnæs, Skjern, Sprogø, and Tystofte* (see Table 4).

*It is interesting to note, that the reference velocity determined at Gedser and Torsminde do not depend on the surface roughness estimate used in the calculations, 0.01 m or 0.001 m. The increase in turbulence for larger roughness is balanced out by a similar decrease of mean velocity. This is a special feature related to the response time of the Dines anemometer used at that particular height (25 m).
6.2 Abild

Abild (1994) contains an analysis of data from the Risø mast, covering the period from 1958 to 1986. The data are wind direction and wind speed at the height 76 m\(^1\), recorded as 10 min averages once an hour. Since almost certainly the largest values will then not be recorded, one would expect that the estimate of the fifty year wind speed would be too low, no matter what statistical method is used. However, based on periods with contiguous 10 min records Abild was able to show that this lead to an approximate 4\% reduction in the maximum wind speeds.

Abild used his version of the WASP technique to obtain the 50-year mean wind speed in eight wind direction sectors and without direction-sector specification at 10 m over sea surface. Since we have used twelve sectors we will only compare the omnidirectional 50-year mean wind speeds.

The roughness over a sea surface is assumed to be given by (Charnock 1955)

\[ z_0 = 0.014 \frac{u^2}{g} \]  

(35)

where \( g \) is the acceleration of gravity.

Abild found the value \( U_{z0} = 28.8 \pm 0.3 \) m/s. The wind speeds influence on this value will be in the interval from 20 to 30 m/s and, using (35) and (37), we see that the corresponding roughness length \( z_0 \) will be about 0.002 m. According to (41) the ratio of the friction velocity over 0.05 m roughness length to the friction velocity over the sea will be about 1.24 so a wind speed of 28.8 m/s at 10 m over the sea will correspond to a wind speed of \( 1.24 \times 28.8 \times \ln(10/0.05)/ \ln(10/0.002) = 22.2 \) m/s. If we ignore our findings at Skjern this value is within one standard deviation from the values we have found for Sprogø, Tystofte and Kegnæs (see Table 4).

\[^{1}\text{The first ten years the measuring height was 72 m.}\]
7 Data Application

In Table 4 we have given, for several sites, the basic wind velocity for each of the twelve wind direction sectors and for all directions together. How is a table of 50-year wind speeds to be applied by engineers in the construction business? First of all, there must be a scheme for converting the values in the table to the basic wind velocity at other heights and over different roughness lengths. Secondly, one should be able to calculate the 50-year event for a construction which does not necessarily have the same response to the wind, i.e., same ratio of load and wind velocity pressure, in all directions. Finally, there must be a method for converting a 50-year event to any T-year event. The relevant method has already been mentioned in section 4.

7.1 Other Heights and Other Roughness Lengths

It is easy to extrapolate the 50-year wind from \( z = 10 \) m to another height \( z' \) if the roughness length \( z_0 \) is kept at the same value 0.05 m. We simply use the logarithmic wind-profile and get

\[
U(z') = \frac{u_*}{\kappa} \left( \ln \left( \frac{z}{z_0} \right) + \ln \left( \frac{z'}{z} \right) \right) = U(z) \left\{ 1 + \frac{\ln(z'/z)}{\ln(z/z_0)} \right\}.
\]  

(36)

However, if the roughness length is changed the situation is not quite that simple because, in order to keep the geostrophic balance, the friction velocity must also change in order for \( G \) to stay constant in (7). It is possible though to take one of the values in Table 4 and predict what the extreme wind \( U_{50}(z'; z'_0) \) will be at another height \( z' \) over another roughness length \( z'_0 \): First we must determine the friction velocity \( u_* \) corresponding to \( U_{50}(z, z_0) \). With \( z = 10 \) m and \( z_0 = 0.05 \) m we get

\[
u_* = \frac{\kappa U(z)}{\ln(z/z_0)}.
\]  

(37)

The relation between the friction velocity \( u'_* \) and \( u_* \) and the roughness length \( z_0 \) and \( z'_0 \), determined by the geostrophic balance equation, can be expressed as

\[
z'_0 s' \sqrt{(\ln(s') - A)^2 + B^2} = z_0 s \sqrt{(\ln(s) - A)^2 + B^2},
\]  

(38)

where

\[(s, s') = \left( \frac{u_*}{fz_0}, \frac{u'_*}{fz'_0} \right).
\]  

(39)

The only unknown in (38) is \( s' \) so we must know how \( h(s) = s \sqrt{(\ln(s) - A)^2 + B^2} \) varies as a function of \( s \). In our case the relevant interval for \( s \) is \([1.6 \times 10^4, 1.7 \times 10^6]\). It turns out that in this interval this function is almost linear and given by

\[
h(s) \approx \tilde{h}(s) = 4.65 \times s^{1.074}.
\]  

(40)

Assuming the simple form (40) we obtain the following relation between the two friction velocities:

\[
u'_* = u_* \left( \frac{z'_0}{z_0} \right)^{1/15}.
\]  

(41)
We note that the friction velocity increases with increasing roughness as expected.

Now we get

\[
U_{50}(z', z_0) = \frac{u_*}{\kappa} \ln \left( \frac{z'}{z_0} \right).
\]  

(42)

It must be emphasized that this procedure in general may be expected to give the correct result in a particular direction sector. Only if the is horizontally homogeneous, with the same roughness length in all directions, will the conditions for predicting the basic wind velocity, irrespective of direction, be fulfilled to the same extent.

### 7.2 Omnidirectional Wind Load

We must now derive a method to determine the T-year event, i.e. the entire wind load on a construction which occurs on average once for every T years. In general it is not reasonable to assume that the site is horizontally homogeneous with the same roughness length in all directions and that the building is axisymmetric. As a consequence the information Table 4 with the basic wind velocity does not suffice. The reason is that we need to know the double exponential probability function for the extreme wind speeds and this, in turn, requires two parameters.

It seems most practical to present tables of \( \alpha \) and \( \tilde{q}(T) \), as defined by the double exponential accumulated probability (46) for the wind load (1) based on the friction velocity \( u_* \). The values are based on a uniform roughness length \( z_0 \) equal to 0.05 m, an air density \( \rho_0 \) equal to 1.25 kg/m\(^3\), and a reference period \( T \) of 50 years.

If we want another reference period \( T' \) we use an equation similar to (21) to determine \( \tilde{q}(T') \):

\[
\tilde{q}(T') = \tilde{q}(T) + \alpha \ln \left( \frac{T'}{T} \right).
\]  

(43)

If the air density \( \rho_0 \) is not 1.25 kg/m\(^3\), but \( \rho \) we must multiply both \( \tilde{q}(T) \) and \( \alpha \) by the ratio \( \rho/\rho_0 \).

As mentioned above the roughness length \( z_0 \) will in general not be equal to 0.05 m but, depending on the direction, be something else \( z'_0 \). This implies that in this particular direction the friction velocity for a given geostrophic wind must, according to (41) be multiplied by \( (z'_0/z_0)^{1/15} \) and, consequently, the wind velocity pressure by \( (z'_0/z_0)^{2/15} \). It follows that both \( \tilde{q}(T) \) and \( \alpha \) must be multiplied by this quantity.

Now we assume that the load \( w_i \) in sector \( i \) is proportional to \( q_i \) (Davenport 1998), but that, as a consequence of lack of axisymmetry, the ratios \( C_i = w_i/q_i \) depend on \( i \).

The basic assumption is then that extreme loads, just like \( q \) have the accumulated probability (17) in each direction sector as well as globally \( (w) \), i.e. irrespective of direction:

\[
P_i(T, w_i) = \exp \left( - \exp \left( - \frac{w_i - \tilde{w}_i}{\beta_i} \right) \right), \quad i = 0, \ldots, 11
\]  

(44)
where
\[
\begin{align*}
\left\{ \begin{array}{c}
\tilde{w}_i \\
\beta_i
\end{array} \right\} = C_i \left\{ \begin{array}{c}
\tilde{q}_i \\
\alpha_i
\end{array} \right\},
\end{align*}
\]
(45)

and
\[
P(T, w) = \exp \left( - \exp \left( - \frac{w - \tilde{w}}{\beta} \right) \right).
\]
(46)

The task is to determine the global parameters \(\tilde{w}\) and \(\beta\).

We now consider two loads \(\mathcal{W}_-\) and \(\mathcal{W}_+\). As pointed out on page 12 the average numbers of times \(\mathcal{N}_-[i]\) and \(\mathcal{N}_+[i]\) that \(\mathcal{W}_-\) and \(\mathcal{W}_+\) are exceeded in sector \(i\) are given by
\[
\mathcal{N}_-[i] = - \ln(P_i(T, \mathcal{W}_-)) = \exp \left( - \frac{\mathcal{W}_- - \tilde{w}_i}{\beta_i} \right)
\]
(47)

and
\[
\mathcal{N}_+[i] = - \ln(P_i(T, \mathcal{W}_+)) = \exp \left( - \frac{\mathcal{W}_+ - \tilde{w}_i}{\beta_i} \right).
\]
(48)

Consequently, the numbers of times \(\mathcal{N}_-\) and \(\mathcal{N}_+\) the levels \(\mathcal{W}_-\) and \(\mathcal{W}_+,\) irrespective of direction, are
\[
\mathcal{N}_- = \sum_{i=0}^{11} \mathcal{N}_-[i]
\]
(49)

and
\[
\mathcal{N}_+ = \sum_{i=0}^{11} \mathcal{N}_+[i].
\]
(50)

According to our basic assumption we have
\[
\mathcal{N}_- = - \ln(P(T, \mathcal{W}_-)) = \exp \left( - \frac{\mathcal{W}_- - \tilde{w}}{\beta} \right)
\]
(51)

and
\[
\mathcal{N}_+ = - \ln(P(T, \mathcal{W}_+)) = \exp \left( - \frac{\mathcal{W}_+ - \tilde{w}}{\beta} \right).
\]
(52)

These two equations can be solved for \(\beta\) and \(\tilde{w}\). The result is
\[
\beta = - \frac{\mathcal{W}_+ - \mathcal{W}_-}{\ln(N_+ + \ln(N_-))} = \frac{\mathcal{W}_+ - \mathcal{W}_-}{\ln(N_-/N_+)}
\]
(53)

and
\[
\tilde{w} = \frac{\mathcal{W}_- + \mathcal{W}_+}{2} - \frac{\beta}{2} \ln(N_- \times N_+).
\]
(54)
We have tested the method on the results from the four sites which we have analyzed: Sprogø, Skjern, Tystofte, and Kegnæs. We assume that \( \varphi_0 \) has its standard value 0.05 m in all directions and that the density \( \rho_0 \) is also equal the standard value 1.25 kg/m\(^3\). Further, we took all the load ratios \( C_i \) to be equal to one. Then we use the method outlined above to calculate, from the values of \( \beta_i = \alpha_i \) and \( \bar{w}_i = \bar{q}_i \), the all-direction values \( \beta \) and \( \bar{w} \). These quantities are also determined directly from the data. The results are shown in the tables 8, 10, 12, and 14. The last line in each table is the result of the calculation. The values of \( W_\tau \) and \( W'_\tau \) are here chosen to be 10\% smaller than the smallest value of \( \bar{q} \) and 10\% larger than the largest value of \( \bar{q} \), respectively.

7.3 Wind Codes

One of the purposes of this investigation is to provide some of the background material for the understanding of the DS 410 (1998) and for Eurocode 1 (1995). In these codes the notation is somewhat different and also the concepts differ from what we have discussed so far. The two subsections below describe basic code definitions and the statistical basis, respectively.

Basic Code Definitions

First of all, the fundamental value of the basic wind velocity in DS 410 (1998) is denoted \( v_{b,0} \) and not \( U_50 \) as we have been using in the previous sections. The corresponding fundamental value of the basic velocity pressure is defined as

\[
q_{b,0} = \frac{1}{2} \rho_0 v_{b,0}^2
\]

where the standard air density \( \rho_0 \) also here is set equal to 1.25 kg/m\(^3\). (The velocity pressure we have been discussing so far in this report has been based on the friction velocity \( u_* \) in order to avoid an explicit reference to the height.)

As demonstrated in Table 5, the basic wind velocity depends on both direction and season. The last may be summarized by (33). In both codes this is taken into account by introducing the basic wind velocity and velocity pressure \( v_b \) and \( q_b \) as functions of wind direction and season by

\[
v_b = c_{\text{dir}} c_{\text{season}} v_{b,0}\]

and

\[
q_b = c_{\text{dir}}^2 c_{\text{season}}^2 q_{b,0}\]

In other words, it is assumed that the variation of the angle is independent of the variation of the season. This can be checked by the data in Table 5. We divide all the numbers in the table by the last column and get \( c_{\text{season}} = v_b/(c_{\text{dir}} v_{b,0}) \). In the new table the numbers, i.e. \( c_{\text{season}} \), in a column should be independent of direction using the code format (56). Instead of a table we display the seasonal variation of \( c_{\text{season}} \) in Fig. 5 and note that the variation within each 61-day period is about 15 to 20\%.

Similarly, if we divide all the numbers in Table 5 by the last row we get \( c_{\text{dir}} = v_b/(c_{\text{season}} v_{b,0}) \) which should be independent of the season. Figure 6 shows that this is also true only within about 15 to 20\%.
The two parameters $c_{\text{dir}}$ and $c_{\text{season}}$ describe climatological variations which are considered representative for the entire country so Figs. 5 and 6 which are derived from Sprogø data must be viewed with some reservation. They are shown here more to explain the concepts in the Wind Codes.

At a specific site the extreme wind load depends on both terrain roughness and topography. In the Wind codes the characteristic mean wind velocity $v_m$\textsuperscript{1} is derived from $v_b$ by

$$v_m(z, D) = c_r(z, D) c_t(z, D) v_b.$$  \hspace{1cm} (58)

\textsuperscript{1}The word 'mean' here is referring to that we are dealing with 10 min mean values. Since the basic wind velocity $v_{b,0}$ is also a 10 min average it would have been logical to include the word 'mean' in this concept too. However, this is not the tradition in the Wind codes.
where the roughness factor \( c_r(z, D) \) and the topography factor \( c_t(z, D) \) account for the variations of \( v_m(z, D) \) due to effective roughness and the topography, respectively.

Here the topography factor will not be considered, viz. \( c_t(z, D) = 1 \) in the following, but, according to the Wind Codes, the roughness factor is given by

\[
c_t = k_t \ln \left( \frac{z}{z_0} \right).
\] (59)

Here \( k_t \) is the terrain factor and this quantity and \( z_0 \) are specified in DS 410 (1998) and Eurocode 1 (1995) for typical terrain categories.

In order to understand these concepts in terms of those developed previously in this report we introduce the following notation:

\[
(z_{0b}, z_b) = (0.05 \text{ m}, 10 \text{ m})
\] (60)

are the basic roughness length and basic height, respectively. The basic friction velocity is defined by means of the basic wind velocity as

\[
u_{*b} = \frac{\kappa z_b}{\ln(z_b/z_{0b})}.
\] (61)

We want to determine \( v_m \) at a particular height \( z \) in a sector where the roughness length is \( z_0 = z_0(D) \). We discussed in subsection 7.1 how a change in roughness length would give rise to a change in the friction velocity for a given geostrophic wind. Changing the roughness from \( z_{0b} \) to \( z_0 \) thus leads to a friction velocity \( u_* \) which can be found by solving the geostrophic balance equation (38) or simply by using the approximation (41)\(^5\) so that

\[
u_* = u_{*b} \left( \frac{z_0}{z_{0b}} \right)^{1/15}.
\] (62)

We have

\[

v_m = \frac{\nu_*}{\kappa} \ln \left( \frac{z}{z_0} \right) \\
= \frac{u_{*b}}{\kappa} \left( \frac{z_0}{z_{0b}} \right)^{1/15} \ln \left( \frac{z}{z_0} \right) \\
= \frac{(z_0/z_{0b})^{1/15}}{\ln(z_b/z_{0b})} \ln \left( \frac{z}{z_0} \right) v_{*b}.
\] (63)

Comparing this result with (58) and (59), we conclude that

\[
k_t = \frac{(z_0/z_{0b})^{1/15}}{\ln(z_b/z_{0b})} \simeq 0.19 \left( \frac{z_0}{z_{0b}} \right)^{1/15}.
\] (64)

\(^5\)The exponent \( 1/15 \simeq 0.067 \) in this equation is determined, as stated in subsection 7.1, as a reasonable fit in one particular interval of the parameter \( u_*/(f z_0) \). Other fits to this exponent have been suggested, e.g. 0.078.
Statistics

The fundamental value of the basic velocity pressure $q_{b,0}$ is the value of the velocity pressure which, at the height $z_b = 10$ m in a uniform terrain with the roughness length $z_0 = 0.05$ m, is exceeded on average once during a period of 50 years.

It is convenient in the following to indicate the reference time $T$ in years by using the notation $\hat{q}_T$ for the velocity pressure which is exceeded once in the period $T$. This means that $\hat{q}_{50} = q_{b,0}$.

We assume that the probability that the level $q$ is not exceeded in the period $T$ is given by (Gumbel 1958)

$$P(T, q) = \exp\left(-e^{(q - \hat{q}_T) / \alpha}\right), \quad (65)$$

where $\alpha$ is a velocity pressure which is independent of $T$.

From (65) we obtain the probability density

$$p(T, q) = \frac{\partial P}{\partial q} = \frac{1}{\alpha} e^{-(q - \hat{q}_T) / \alpha} \exp\left(-e^{(q - \hat{q}_T) / \alpha}\right), \quad (66)$$

The mean and the average of $q$ are given by

$$\langle q \rangle_T = \int_0^\infty q p(T, q) dq \approx \int_{-\infty}^\infty q p(T, q) dq = \hat{q}_T + \gamma \alpha, \quad (67)$$

where $\gamma = 0.57721\ldots$ is Euler’s constant, and

$$\sigma_T^2 = \int_0^\infty (q - \langle q \rangle_T)^2 p(T, q) dq \approx \int_{-\infty}^\infty (q - \langle q \rangle_T)^2 p(T, q) dq = \frac{\pi^2}{6} \alpha^2. \quad (68)$$

Figure 7 shows the probability density $p(T, q)$ with indications of the magnitudes of $\hat{q}_T$, $\langle q \rangle_T$, and $\sigma_T$.

Under the assumption that the individual excursions of the velocity pressure beyond $q$ are statistically independent, the probability for a particular number of excursions is a Poisson distribution with the mean number of excursions $N$ given by

$$N(T, q) = \exp\left(-\frac{q - \hat{q}_T}{\alpha}\right). \quad (69)$$

This number must of course be proportional to the observation time $T$, i.e.

$$\frac{N(T, q)}{T} = \frac{N(T', q)}{T'}. \quad (70)$$

Inserting (69), we obtain the relation between $\hat{q}_{T'}$ and $\hat{q}_T$

$$\hat{q}_{T'} = \hat{q}_T + \alpha \ln \left(\frac{T'}{T}\right). \quad (71)$$

The probability $P_n(T, q)$ for $n$ excursions beyond $q$ in the time interval $T$ is then

$$P_n(T, q) = \frac{N^n(T, q)}{n!} e^{-N(T, q)}. \quad (72)$$
Specifically, the probability for no excursions becomes

\[
T_0(T, q) = e^{-N(T, q)} = P(T, q),
\]

which is consistent with the definition (65).

The probability that the level \( q \) is exceeded at least once during the period \( T \) is

\[
\Phi(T, q) = 1 - P(T, q) = 1 - \exp \left(-e^{(q - \bar{q}_T) / \alpha} \right).
\]

Now we can also ask for the velocity pressure \( q(T, \Phi) \) which during the time \( T \) is exceeded at least once with the probability \( \Phi \). The answer to this question is obtained by solving (74) with respect to \( q \). The solution is

\[
q(T, \Phi) = \bar{q}_T - \alpha \ln(-\ln(1 - \Phi)).
\]

In particular, when \( T = 1 \) year we get

\[
q(1, \Phi) = \bar{q}_1 \left(1 - K \ln(-\ln(1 - \Phi)) \right),
\]

where we follow the notation in the Wind Codes and define \( K = \alpha / \bar{q}_1 \).

We want to express \( q(1, \Phi) \) in terms of the basic velocity pressure \( \bar{q}_{50} = q_{b,0} \) and use the relation (71) to obtain

\[
\bar{q}_1 = \frac{\bar{q}_{50}}{1 + K \ln(50)}.
\]

Inserting in (76), we get

\[
q(1, \Phi) = \frac{1 - K \ln(-\ln(1 - \Phi)) \bar{q}_{50}}{1 + K \ln(50)}.
\]
This equation is slightly different from the formulation in the Wind Codes. It is argued, based on the definition that \( \tilde{q}_{50} \) is the velocity pressure which is on average exceeded once in the period of 50 years, that the average number of times this velocity pressure is exceeded in one year is \( 1/50 \). The probability that \( \tilde{q}_{50} \) is exceeded at least once in one year is consequently

\[
\overline{P}(1, \tilde{q}_{50}) = 1 - \exp\left(-\frac{1}{50}\right) \approx 1 - \left(1 - \frac{1}{50}\right) = \frac{1}{50}.
\] (79)

On the other hand, we have from (74)

\[
\overline{P}(1, \tilde{q}_{50}) = 1 - \exp\left(-e^{-(\tilde{q}_{50} - \tilde{q}_1)}\right)
\] (80)

and, comparing these two equations, the following relation allows provides an alternative to (77)

\[
\tilde{q}_1 \approx \frac{\tilde{q}_{50}}{1 - K \ln(-\ln(0.98))}.
\] (81)

Using this expression we obtain the same approximate equation as in the Wind Codes, namely

\[
q(1, \overline{P}) = \frac{1 - K \ln(-\ln(1 - \overline{P}))}{1 - K \ln(-\ln(0.98))} \tilde{q}_{50}.
\] (82)

Note that the difference between (82) and (78) is insignificant from a practical point of view since \( \ln(50) \approx 3.91 \) whereas \( -\ln(-\ln(0.98)) \approx 3.90 \).
8 Conclusions

We have analyzed wind data from four sites, Skjern, Kegnæs, Sprogø, and Tystofte, in order to obtain estimates—in twelve wind sectors and overall—of the basic wind velocity which is defined as the average ten-minute wind speed at the altitude 10 m in a homogeneous terrain with the roughness length 0.05 m which, on average, is exceeded once in a period of fifty years. The observed data were transformed to standard friction velocity $u_*$ over 0.05 m roughness by means of the WA³P technique (Mortensen et al. 1993). The atmospheric surface stratification was assumed neutral because only high wind speeds are of interest. The extreme values of the wind pressure based on $u_*$ and an air density of 1.25 kg/m$^3$ was used to determine the basic wind velocity.

The general result of this analysis is shown in Table 4. The overall basic wind velocity seems to vary from west to east with 25 m/s at Skjern to about 21 m/s at the minimum at Sprogø and then to 22 m/s at Tystofte. This is consistent with the findings by Jensen & Frank (1970). Based on their measurements we can infer that at Torsminde at the west coast of Jutland near Limfjorden the basic wind velocity is 25 m/s and 23 m/s at Geoløk at the south tip of the island Falster south of Zealand. Also Abild's (1994) results are consistent with out data at Sprogø, Tystofte and Kegnæs.

The conclusion is that there is a weakening of the basic wind velocity when moving from west to east. This is supported by another investigation by Mortensen et al. (1999). A map of mean of the square of the ten-minute wind speed at 10 m altitude over 0.05 m roughness length was produced as a by-product and is shown in Fig. 8. Figure 8 indicates that the wind is significantly stronger at the north-west part of Jutland than anywhere else in Denmark.

We might therefore conclude that west of a line going from Esbjerg to the island Læsø in the Kattegat Sea it is reasonable to recommend a basic wind velocity of 25 m/s while it should be 23 m/s in the rest of the country.

We found that the basic wind velocity has a significant variation with direction as well as season. Table 5 and Fig. 6 show that, irrespective of season, the highest basic wind velocities are found in the western sectors, from about 210° to about 330°. There is also a pronounced variation with season: the basic wind velocity is largest in the winter and smallest in the summer. Based on the data from Sprogø it was found that the variation of the basic wind velocity from winter to summer was about 30%. The data from the four sites do not show exactly the same seasonal variation. As Fig. 9 shows, it is largest at Skjern and Kegnæs.

One of the reasons that the seasonal variations seem more dependent of direction at Skjern and Kegnæs is probably that the measuring heights at these sites, 23.8 m and 23.4 m, respectively, are smaller than at Sprogø (70 m) and Tystofte (39.3 m). This is so because an uncertainty in the roughness length will, for a given wind speed, lead to a larger uncertainty in the determination of the friction velocity the smaller the measuring altitude. With the following line arguments supports this postulate.

Let the wind profile be given by

$$U(z) = \frac{u_*}{\kappa} \ln \left( \frac{z}{z_0} \right).$$

(83)
Figure 8. Contour plot of the average square wind speed.

Figure 9. Seasonal variation of the basic wind velocity at Skjern, Sprogø, Tystofte and Kegnæs in all twelve direction sectors. The bullets are the western sectors, the diamonds the rest.
Then, with $U(z)$ fixed, we get by logarithmic differentiation

$$\frac{\delta u_*}{u_*} = \frac{1}{\ln(z/z_0)} \frac{\delta z_0}{z_0}.$$  \hspace{1cm} (84)

This equation shows, at least qualitatively, that the relative uncertainty in $u_*$ for a given uncertainty in $z_0$ is a decreasing function of height. Exactly how important the effect is must be evaluated in each particular case.

In section 7 we showed how the design wind velocities can be determined at other altitudes than 10 m and in situations where the roughness lengths are different from 0.05 m. We also showed how easy it is—under the assumption that the extreme winds in any two direction sectors are statistically independent—to integrate the wind load over all directions in cases where the structure and/or the terrain in question are not axisymmetric.
Acknowledgements

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References


A Detailed results

A.1 Sprogø

Figure 10. Sprogø. The $M = 121$ ordered values $q[m]$ plotted against $-\ln(-\ln(m/(M + 1)))$ for each direction sector. The least-square fits are shown. The vertical line in each plot corresponds to $-\ln(-\ln(m/(M + 1))) = 0$.

Figure 11. Sprogø. The $M = 121$ ordered values $q[m]$ plotted against $-\ln(-\ln(m/(M + 1)))$ irrespective of wind direction.
Figure 12. Sprog. The basic wind velocity, based on 61-day periods, in the 12 direction sectors (thick line) and ± the standard deviation (thin line).

\[ U_{50} \]

\[ T_0 = 61 \text{ days} \]

\[ D \]

\[ q[m] \]

\[ -\ln \left( -\ln \left( \frac{m}{M+1} \right) \right) \]

Figure 13. Sprog. The M = 20 ordered values \( q[m] \) plotted against \( -\ln(-\ln(m/(M + 1))) \) for each direction sector. The least-square fits are shown. The vertical line in each plot corresponds to \( -\ln(-\ln(m/(M + 1))) = \theta. \)
Figure 14. Sprogø. The $M = 20$ ordered values $q[m]$ plotted against $-\ln(-\ln(m/(M + 1)))$ irrespective of wind direction.

Figure 15. Sprogø. The basic wind velocity, based on one-year periods, in the 12 direction sectors (thick line) and ± the standard deviation (thin line).
Table 7. The basic wind velocity $U_{50}$ with standard deviations at the height $z = 10$ m over the roughness length $z_0 = 0.05$ m, based on data from Sprogø. Two basic values of $T_0$ has been used.

<table>
<thead>
<tr>
<th>$D$</th>
<th>$T_0 = 61$ days $U_{50}$ (m/s)</th>
<th>$T_0 = 1$ year $U_{50}$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>000°</td>
<td>14.9 ± 0.5</td>
<td>15.1 ± 1.3</td>
</tr>
<tr>
<td>030°</td>
<td>14.2 ± 0.5</td>
<td>14.2 ± 1.3</td>
</tr>
<tr>
<td>060°</td>
<td>15.2 ± 0.5</td>
<td>14.2 ± 1.2</td>
</tr>
<tr>
<td>090°</td>
<td>16.5 ± 0.5</td>
<td>15.9 ± 1.3</td>
</tr>
<tr>
<td>120°</td>
<td>15.6 ± 0.5</td>
<td>14.6 ± 1.0</td>
</tr>
<tr>
<td>150°</td>
<td>16.6 ± 0.5</td>
<td>16.3 ± 1.3</td>
</tr>
<tr>
<td>180°</td>
<td>18.2 ± 0.6</td>
<td>16.5 ± 1.2</td>
</tr>
<tr>
<td>210°</td>
<td>18.6 ± 0.6</td>
<td>18.1 ± 1.3</td>
</tr>
<tr>
<td>240°</td>
<td>19.0 ± 0.6</td>
<td>18.6 ± 1.2</td>
</tr>
<tr>
<td>270°</td>
<td>19.8 ± 0.6</td>
<td>20.0 ± 1.5</td>
</tr>
<tr>
<td>300°</td>
<td>18.7 ± 0.6</td>
<td>17.8 ± 1.3</td>
</tr>
<tr>
<td>330°</td>
<td>15.9 ± 0.5</td>
<td>15.6 ± 1.2</td>
</tr>
<tr>
<td>000°–360°</td>
<td>20.7 ± 0.6</td>
<td>20.4 ± 1.3</td>
</tr>
</tbody>
</table>

Table 8. Sprogø. $\alpha$ and $\tilde{q}$ based on the friction velocity under standard conditions: $z_0 = 0.05$ m, $\rho_0 = 1.25$ kg/m$^3$, $T = 50$ years, and $T_0 = 61$ days. The last row shows calculated values of $\alpha$ and $\tilde{q}$ in accordance with the equations in subsection 7.2.

<table>
<thead>
<tr>
<th>$D$</th>
<th>$\alpha$</th>
<th>$\tilde{q}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>000°</td>
<td>0.102 ± 0.009</td>
<td>0.791 ± 0.053</td>
</tr>
<tr>
<td>030°</td>
<td>0.098 ± 0.009</td>
<td>0.718 ± 0.051</td>
</tr>
<tr>
<td>060°</td>
<td>0.110 ± 0.010</td>
<td>0.822 ± 0.058</td>
</tr>
<tr>
<td>090°</td>
<td>0.119 ± 0.011</td>
<td>0.976 ± 0.063</td>
</tr>
<tr>
<td>120°</td>
<td>0.100 ± 0.009</td>
<td>0.870 ± 0.053</td>
</tr>
<tr>
<td>150°</td>
<td>0.122 ± 0.011</td>
<td>0.979 ± 0.064</td>
</tr>
<tr>
<td>180°</td>
<td>0.146 ± 0.014</td>
<td>1.180 ± 0.077</td>
</tr>
<tr>
<td>210°</td>
<td>0.148 ± 0.014</td>
<td>1.232 ± 0.078</td>
</tr>
<tr>
<td>240°</td>
<td>0.156 ± 0.014</td>
<td>1.288 ± 0.082</td>
</tr>
<tr>
<td>270°</td>
<td>0.168 ± 0.015</td>
<td>1.401 ± 0.088</td>
</tr>
<tr>
<td>300°</td>
<td>0.155 ± 0.014</td>
<td>1.251 ± 0.081</td>
</tr>
<tr>
<td>330°</td>
<td>0.112 ± 0.010</td>
<td>0.902 ± 0.059</td>
</tr>
<tr>
<td>000°–330°</td>
<td>0.170 ± 0.016</td>
<td>1.525 ± 0.089</td>
</tr>
<tr>
<td>000°–330°</td>
<td>0.154</td>
<td>1.547</td>
</tr>
</tbody>
</table>
A.2 Skjern

\[ q[m] \]

\[ -\ln\left(-\ln\left(\frac{m}{M+1}\right)\right) \]

*Figure 16. Skjern. The \( M = 24 \) ordered values \( q[m] \) plotted against \(-\ln(-\ln(m/(M+1)))\) for each direction sector. The least-square fits are shown. The vertical line in each plot corresponds to \(-\ln(-\ln(m/(M+1))) = 0\).*

\[ x = -\ln(-\ln(P)) \]

*Figure 17. Skjern. The \( M = 24 \) ordered values \( q[m] \) plotted against \(-\ln(-\ln(m/(M+1)))\) irrespective of wind direction.*
Figure 18. Skjern. The basic wind velocity, based on 61-day periods, in the 12 direction sectors (thick line) and ± the standard deviation (thin line).

Figure 19. Skjern. The $M = 15$ ordered values $q[m]$ plotted against $-\ln(-\ln(m/(M + 1)))$ for each direction sector. The least-square fits are shown. The vertical line in each plot corresponds to $-\ln(-\ln(m/(M + 1))) = 0$. 
Figure 20. Skjern. The $M = 15$ ordered values $q[m]$ plotted against $-\ln(-\ln(m/(M + 1)))$ irrespective of wind direction.

Figure 21. Skjern. The basic wind velocity, based on one-year periods, in the 12 direction sectors (thick line) and ± the standard deviation (thin line).
Table 9. The basic wind velocity $U_{50}$ with standard deviations at the height $z = 10$ m over the roughness length $z_0 = 0.05$ m, based on data from Skjern. Two basic values of $T_0$ has been used.

<table>
<thead>
<tr>
<th>$D$</th>
<th>$T_0 = 61$ days $U_{50}$ (m/s)</th>
<th>$T_0 = 1$ year $U_{50}$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>000°</td>
<td>16.1 ± 0.6</td>
<td>15.8 ± 1.6</td>
</tr>
<tr>
<td>030°</td>
<td>17.1 ± 0.7</td>
<td>17.3 ± 1.8</td>
</tr>
<tr>
<td>060°</td>
<td>17.4 ± 0.7</td>
<td>18.0 ± 1.9</td>
</tr>
<tr>
<td>090°</td>
<td>17.7 ± 0.7</td>
<td>16.5 ± 1.6</td>
</tr>
<tr>
<td>120°</td>
<td>16.8 ± 0.6</td>
<td>16.0 ± 1.4</td>
</tr>
<tr>
<td>150°</td>
<td>18.9 ± 0.7</td>
<td>19.4 ± 1.8</td>
</tr>
<tr>
<td>180°</td>
<td>20.1 ± 0.8</td>
<td>20.3 ± 1.8</td>
</tr>
<tr>
<td>210°</td>
<td>20.7 ± 0.8</td>
<td>24.5 ± 2.5</td>
</tr>
<tr>
<td>240°</td>
<td>20.9 ± 0.8</td>
<td>24.8 ± 2.6</td>
</tr>
<tr>
<td>270°</td>
<td>22.1 ± 0.8</td>
<td>23.4 ± 2.4</td>
</tr>
<tr>
<td>300°</td>
<td>23.9 ± 0.9</td>
<td>25.9 ± 2.7</td>
</tr>
<tr>
<td>330°</td>
<td>20.3 ± 0.8</td>
<td>21.8 ± 2.2</td>
</tr>
<tr>
<td>000°–360°</td>
<td>24.9 ± 0.9</td>
<td>28.7 ± 2.5</td>
</tr>
</tbody>
</table>

Table 10. Skjern, $\alpha$ and $\tilde{q}$ based on the friction velocity under standard conditions: $z_0 = 0.05$ m, $\rho_0 = 1.25$ kg/m$^3$, $T = 50$ years, and $T_0 = 61$ days. The last row shows calculated values of $\alpha$ and $\tilde{q}$ in accordance with the equations in subsection 7.2.

<table>
<thead>
<tr>
<th>$D$</th>
<th>$\alpha$</th>
<th>$\tilde{q}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>000°</td>
<td>0.123 ± 0.013</td>
<td>0.921 ± 0.073</td>
</tr>
<tr>
<td>030°</td>
<td>0.140 ± 0.015</td>
<td>1.044 ± 0.084</td>
</tr>
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<td>060°</td>
<td>0.139 ± 0.015</td>
<td>1.078 ± 0.083</td>
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<tr>
<td>090°</td>
<td>0.141 ± 0.015</td>
<td>1.113 ± 0.084</td>
</tr>
<tr>
<td>120°</td>
<td>0.118 ± 0.013</td>
<td>1.005 ± 0.071</td>
</tr>
<tr>
<td>150°</td>
<td>0.167 ± 0.018</td>
<td>1.266 ± 0.101</td>
</tr>
<tr>
<td>180°</td>
<td>0.186 ± 0.020</td>
<td>1.445 ± 0.112</td>
</tr>
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<td>210°</td>
<td>0.192 ± 0.020</td>
<td>1.533 ± 0.116</td>
</tr>
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<td>240°</td>
<td>0.195 ± 0.021</td>
<td>1.554 ± 0.119</td>
</tr>
<tr>
<td>270°</td>
<td>0.216 ± 0.023</td>
<td>1.744 ± 0.130</td>
</tr>
<tr>
<td>300°</td>
<td>0.259 ± 0.027</td>
<td>2.033 ± 0.156</td>
</tr>
<tr>
<td>330°</td>
<td>0.191 ± 0.020</td>
<td>1.462 ± 0.114</td>
</tr>
<tr>
<td>000°–330°</td>
<td>0.263 ± 0.028</td>
<td>2.209 ± 0.159</td>
</tr>
<tr>
<td>000°–330°</td>
<td>0.226</td>
<td>2.151</td>
</tr>
</tbody>
</table>
A.3 Tystofte

- $D = 0^\circ$
- $D = 90^\circ$
- $D = 180^\circ$
- $D = 270^\circ$
- $D = 360^\circ$

$q[m]$

Figure 22. Tystofte. The $M = 93$ ordered values $q[m]$ plotted against $-\ln(-\ln(m/(M + 1)))$ for each direction sector. The least-square fits are shown. The vertical line in each plot corresponds to $-\ln(-\ln(m/(M + 1))) = 0$.

$x = -\ln(-\ln(P))$

Figure 23. Tystofte. The $M = 93$ ordered values $q[m]$ plotted against $-\ln(-\ln(m/(M + 1)))$ irrespective of wind direction.
Figure 24. Tystofte. The basic wind velocity, based on 61-day periods, in the 12 direction sectors (thick line) and ± the standard deviation (thin line).

Figure 25. Tystofte. The $M = 15$ ordered values $q[m]$ plotted against $-\ln(-\ln(m/(M+1)))$ for each direction sector. The least-square fits are shown. The vertical line in each plot corresponds to $-\ln(-\ln(m/(M+1))) = 0$. 

$U_{50}$

$T_0 = 61$ days

$D$

$-\ln\left(-\ln\left(\frac{m}{M+1}\right)\right)$
Figure 26. Tystofte. The $M = 15$ ordered values $q[m]$ plotted against $-\ln(-\ln(m/(M+1)))$ irrespective of wind direction.

Figure 27. Tystofte. The basic wind velocity, based on one-year periods, in the 12 direction sectors (thick line) and ± the standard deviation (thin line).
Table 11. The basic wind velocity $U_{50}$ with standard deviations at the height $z = 10$ m over the roughness length $z_0 = 0.05$ m, based on data from Tysstofte. Two basic values of $T_0$ has been used.

<table>
<thead>
<tr>
<th>$D$</th>
<th>$T_0 = 61$ days $U_{50}$ (m/s)</th>
<th>$T_0 = 1$ year $U_{50}$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>000°</td>
<td>18.0 ± 0.7</td>
<td>17.3 ± 2.0</td>
</tr>
<tr>
<td>030°</td>
<td>16.0 ± 0.7</td>
<td>16.1 ± 1.8</td>
</tr>
<tr>
<td>060°</td>
<td>19.1 ± 0.8</td>
<td>19.3 ± 2.0</td>
</tr>
<tr>
<td>090°</td>
<td>19.3 ± 0.7</td>
<td>18.0 ± 1.7</td>
</tr>
<tr>
<td>120°</td>
<td>15.8 ± 0.6</td>
<td>15.9 ± 1.4</td>
</tr>
<tr>
<td>150°</td>
<td>17.7 ± 0.7</td>
<td>18.2 ± 1.8</td>
</tr>
<tr>
<td>180°</td>
<td>19.2 ± 0.7</td>
<td>18.6 ± 1.7</td>
</tr>
<tr>
<td>210°</td>
<td>19.5 ± 0.7</td>
<td>20.3 ± 1.8</td>
</tr>
<tr>
<td>240°</td>
<td>21.1 ± 0.8</td>
<td>22.5 ± 2.1</td>
</tr>
<tr>
<td>270°</td>
<td>21.4 ± 0.8</td>
<td>21.6 ± 1.9</td>
</tr>
<tr>
<td>300°</td>
<td>20.8 ± 0.8</td>
<td>21.1 ± 1.9</td>
</tr>
<tr>
<td>330°</td>
<td>18.8 ± 0.7</td>
<td>18.3 ± 1.7</td>
</tr>
<tr>
<td>000°–360°</td>
<td>22.6 ± 0.8</td>
<td>24.1 ± 1.9</td>
</tr>
</tbody>
</table>

Table 12. Tysstofte. $\alpha$ and $\bar{q}$ based on the friction velocity under standard conditions: $z_0 = 0.05$ m, $\rho_0 = 1.25$ kg/m$^3$, $T = 50$ years, and $T_0 = 61$ days. The last row shows calculated values of $\alpha$ and $\bar{q}$ in accordance with the equations in subsection 7.2.

<table>
<thead>
<tr>
<th>$D$</th>
<th>$\alpha$</th>
<th>$\bar{q}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>000°</td>
<td>0.155 ± 0.016</td>
<td>1.154 ± 0.003</td>
</tr>
<tr>
<td>030°</td>
<td>0.124 ± 0.013</td>
<td>0.917 ± 0.075</td>
</tr>
<tr>
<td>060°</td>
<td>0.173 ± 0.018</td>
<td>1.294 ± 0.104</td>
</tr>
<tr>
<td>090°</td>
<td>0.165 ± 0.018</td>
<td>1.324 ± 0.100</td>
</tr>
<tr>
<td>120°</td>
<td>0.105 ± 0.011</td>
<td>0.892 ± 0.063</td>
</tr>
<tr>
<td>150°</td>
<td>0.147 ± 0.016</td>
<td>1.112 ± 0.089</td>
</tr>
<tr>
<td>180°</td>
<td>0.165 ± 0.017</td>
<td>1.315 ± 0.099</td>
</tr>
<tr>
<td>210°</td>
<td>0.163 ± 0.017</td>
<td>1.356 ± 0.098</td>
</tr>
<tr>
<td>240°</td>
<td>0.190 ± 0.021</td>
<td>1.589 ± 0.120</td>
</tr>
<tr>
<td>270°</td>
<td>0.204 ± 0.022</td>
<td>1.632 ± 0.123</td>
</tr>
<tr>
<td>300°</td>
<td>0.192 ± 0.020</td>
<td>1.542 ± 0.115</td>
</tr>
<tr>
<td>330°</td>
<td>0.163 ± 0.017</td>
<td>1.265 ± 0.099</td>
</tr>
<tr>
<td>000°–330°</td>
<td>0.205 ± 0.022</td>
<td>1.827 ± 0.123</td>
</tr>
<tr>
<td>000°–360°</td>
<td>0.186</td>
<td>1.859</td>
</tr>
</tbody>
</table>
A.4 Kegnæs

$\begin{align*}
D = 000^\circ & \quad D = 030^\circ & \quad D = 060^\circ \\
D = 090^\circ & \quad D = 120^\circ & \quad D = 150^\circ \\
D = 180^\circ & \quad D = 210^\circ & \quad D = 240^\circ \\
D = 270^\circ & \quad D = 300^\circ & \quad D = 330^\circ
\end{align*}$

$q[m]$

$-\ln\left(-\ln\left(\frac{m}{M+1}\right)\right)$

Figure 28. Kegnæs. The $M = 42$ ordered values $q[m]$ plotted against $-\ln(-\ln(m/(M + 1)))$ for each direction sector. The least-square fits are shown. The vertical line in each plot corresponds to $-\ln(-\ln(m/(M + 1))) = 0$.

$x = -\ln(-\ln(P))$

Figure 29. Kegnæs. The $M = 42$ ordered values $q[m]$ plotted against $-\ln(-\ln(m/(M + 1)))$ irrespective of wind direction.
Figure 30. Kegn. The basic wind velocity, based on 61-day periods, in the 12 direction sectors (thick line) and ± the standard deviation (thin line).

Table 13. The basic wind velocity $U_{50}$ with standard deviations at the height $z = 10$ m over the roughness length $z_0 = 0.05$ m, based on data from Kegn. One basic value of $T_0=61$ days has been used.

<table>
<thead>
<tr>
<th>$D$</th>
<th>$T_0=61$ days $U_{50}$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$000^\circ$</td>
<td>15.9 ± 1.0</td>
</tr>
<tr>
<td>$030^\circ$</td>
<td>14.3 ± 0.8</td>
</tr>
<tr>
<td>$060^\circ$</td>
<td>16.5 ± 0.9</td>
</tr>
<tr>
<td>$090^\circ$</td>
<td>16.7 ± 1.0</td>
</tr>
<tr>
<td>$120^\circ$</td>
<td>15.3 ± 0.9</td>
</tr>
<tr>
<td>$150^\circ$</td>
<td>14.5 ± 0.9</td>
</tr>
<tr>
<td>$180^\circ$</td>
<td>17.6 ± 1.1</td>
</tr>
<tr>
<td>$210^\circ$</td>
<td>20.1 ± 1.2</td>
</tr>
<tr>
<td>$240^\circ$</td>
<td>19.9 ± 1.1</td>
</tr>
<tr>
<td>$270^\circ$</td>
<td>21.0 ± 1.2</td>
</tr>
<tr>
<td>$300^\circ$</td>
<td>16.2 ± 0.9</td>
</tr>
<tr>
<td>$330^\circ$</td>
<td>15.5 ± 1.0</td>
</tr>
<tr>
<td>$000^\circ$–$360^\circ$</td>
<td>21.7 ± 1.2</td>
</tr>
</tbody>
</table>
Table 14. Kgs/m, α and \( \bar{\tau} \) based on the friction velocity under standard conditions: \( z_0 = 0.05 \text{ m}, \rho_0=1.25 \text{ kg/m}^3, T = 50 \text{ years}, \) and \( T_0 = 61 \text{ days} \). The last row shows calculated values of α and \( \bar{\tau} \) in accordance with the equations in subsection 7.2.

<table>
<thead>
<tr>
<th>( D )</th>
<th>α</th>
<th>( \bar{\tau} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>000°</td>
<td>0.121</td>
<td>0.808 ± 0.109</td>
</tr>
<tr>
<td>030°</td>
<td>0.094</td>
<td>0.731 ± 0.086</td>
</tr>
<tr>
<td>060°</td>
<td>0.122</td>
<td>0.906 ± 0.111</td>
</tr>
<tr>
<td>090°</td>
<td>0.125</td>
<td>0.901 ± 0.113</td>
</tr>
<tr>
<td>120°</td>
<td>0.108</td>
<td>0.830 ± 0.099</td>
</tr>
<tr>
<td>150°</td>
<td>0.097</td>
<td>0.748 ± 0.088</td>
</tr>
<tr>
<td>180°</td>
<td>0.145</td>
<td>1.103 ± 0.133</td>
</tr>
<tr>
<td>210°</td>
<td>0.187</td>
<td>1.440 ± 0.171</td>
</tr>
<tr>
<td>240°</td>
<td>0.174</td>
<td>1.406 ± 0.160</td>
</tr>
<tr>
<td>270°</td>
<td>0.191</td>
<td>1.573 ± 0.175</td>
</tr>
<tr>
<td>300°</td>
<td>0.117</td>
<td>0.933 ± 0.108</td>
</tr>
<tr>
<td>330°</td>
<td>0.115</td>
<td>0.857 ± 0.106</td>
</tr>
<tr>
<td>000°-330°</td>
<td>0.196</td>
<td>1.673 ± 0.179</td>
</tr>
<tr>
<td>000°-330°</td>
<td>0.179</td>
<td>1.700</td>
</tr>
</tbody>
</table>
Wind-speed data from four sites in Denmark have been analyzed in order to obtain estimates of the basic wind velocity which is defined as the 50-year wind speed under standard conditions, i.e., ten-minute averages at the height 10 m over a uniform terrain with the roughness length 0.05 m. The sites are, from west, Skjern (15 years), Kegnæs (7 years), Sprogø (20 years), and Tystofte (15 years). The data are ten minute averages of wind speed, wind direction, temperature and pressure. The last two quantities are used to determine the air density $\rho$. The data are cleaned for terrain effects by means of a slightly modified WAP technique where the sector speed-up factors and roughness lengths are linearly smoothed with a direction resolution of one degree. Assuming geostrophic balance, all the wind-velocity data are transformed to friction velocity $u_*$ and direction at standard conditions by means of the geostrophic drag law for neutral stratification. The basic wind velocity in 30° sectors are obtained through ranking of the largest values of the friction velocity pressure $1/2pu_*^2$ taken both once every two months and once every year. The main conclusion is that the basic wind velocity is significantly larger at Skjern, close to the west coast of Jutland, than at any of the other sites. Irrespective of direction, the present standard estimates of 50-year wind are $25 \pm 1$ m/s at Skjern and $22 \pm 1$ m/s at the other three sites. These results are in agreement with those obtained by Jensen & Franck (1970) and Abild (1994) and supports the conclusion that the wind climate at the west coast of Jutland is more extreme than in any other part of the country. Simple procedures to translate in a particular direction sector the standard basic wind velocity to conditions with a different roughness length and height are presented. It is shown that a simple scheme makes it possible to calculate the total 50-year extreme load on a general structure without symmetry in an inhomogeneous terrain. A special section is devoted to the interpretation of the concepts in the Danish wind code DS 410 (1998) and Eurocode 1 (1995).