DTU Library

The Boat and Barge Routing Problem

Gamst, Mette; Kjeldsen, Niels

Published in:
Proceedings

Publication date:
2012

Document Version
Publisher's PDF, also known as Version of record

The Boat and Barge Routing Problem ⋆

Mette Gamst [a] and Niels Kjeldsen [a,b]

[a] *University of Southern Denmark, Campusvej 55, DK-5230 Odense M, Denmark*

[b] *DONG Energy, Kraftvaerksvej 53, DK-7000 Fredericia, Denmark*

Key words: BBRP, VRPTW, TTRP, column generation

1 Introduction

The thermal power plants in Denmark need to be refueled at regular intervals to ensure stable and reliable production of electricity. Fuel (typically coal) for the power plants is delivered from overseas to two central depots in Denmark, from here the fuel is distributed to the power plants by an internal fleet of tug boats and barges. The fuel is loaded into a barge which is then pulled by a tug boat to a power plant harbor where the barge is unloaded. During the loading and unloading period the tug boat is not needed and can service other deliveries. Barges cannot move on their own. The internal fleet does not always have the capacity to service all power plants, in these cases an external delivery can be made at a significantly higher cost.

The Barge and Boat Routing Problem (BBRP) shares similarities with the Vehicle Routing Problem with Time Windows (VRPTW), see e.g. Toth and Vigo [6], and with the Truck and Trailer Routing Problem (TTRP), see e.g. Chao [2].

As in the VRPTW each delivery is also associated with a time window. The key difference from the TTRP are the autonomous vehicles in the BBRP, i.e., the possibility to change which tug boat pulls which barge — in the TTRP trailers are always moved by the same truck. Drexl considers a variant of the Vehicle Routing Problem in [3] called the Vehicle Routing Problem with Trailers and Transshipments (VRPTT). The VRPTT also includes autonomous vehicles along with other complicating constraints. Drexl solves a simplified core part of the problem with a branch-and-cut algorithm, but only for a very small instance. He concludes that the addition of autonomous vehicles greatly increases the complexity of the problem.

⋆ This project was supported in part by DONG Energy and by the Villum-Kann-Rasmussen Foundation.

The contribution of this work is the introduction of the BBRP, a MIP formulation (which we do not state here) and a branch-and-price algorithm for the BBRP. The mathematical formulation is based on formulations for the VRPTW, see e.g. Kallehauge et al. [4]. We propose a branch-and-price algorithm with two pricing problems: one for generating barge paths and one for generating tug boat paths. Both the mathematical formulation and the branch-and-price algorithm have been tested on a number of real-life test instances based on data from the Danish utility company DONG Energy. Early results indicate that the branch-and-price algorithm clearly outperforms the mathematical formulation.

2 Branch-and-price algorithm

The BBRP is Dantzig-Wolfe decomposed and solved to optimality using a branch-and-price (BP) algorithm. Two different pricing problems are generated; one for creating paths for tug boats and one for creating paths for barges. The master problem merges the paths into an overall feasible solution. The BBRP works on a graph $G = (N, A)$. The nodes N is the union of depots (denoted D) and deliveries at power plants (denoted L); arcs A connect the nodes. Time is discretized into a set of time stamps T. Let k_B be the set of barges, k_T be the set of tug boats and let the set of paths for vehicles of type $k \in \{k_B, k_T\}$ be denoted P_k. The binary variable z_i is an expensive, external delivery to power plant i, with cost c_i. Let y_p^k be a binary variable denoting whether or not a vehicle k travels on path p, let $c_p^k \geq 0$ be the cost for using a vehicle of type k on path p. The master problem is:

$$\min \quad \sum_{p \in P_{k_T}} c_p^{k_T} y_p^{k_T} + \sum_{i \in L} c_i z_i \tag{1}$$

$$\text{s.t.} \quad \sum_{p \in P_{k_B}} \delta_p^i y_p^{k_B} + z_i = 1 \qquad \forall i \in L \tag{2}$$

$$\sum_{p \in P_{k_B}} \delta_{ij\tau}^p y_p^{k_B} - \sum_{p \in P_{k_T}} \delta_{ij\tau}^p y_p^{k_T} \leq 0 \qquad \forall (i,j) \in A, \forall \tau \in T \tag{3}$$

$$\sum_{p \in P_k} y_p^k \leq |k| \qquad k \in \{k_B, k_T\} \tag{4}$$

$$y_p^k \in \{0, 1\} \qquad \forall p \in P_k, \forall k \in K \tag{5}$$

$$z_i \in \{0, 1\} \qquad \forall i \in L \tag{6}$$

The objective function (1) minimizes the total cost of transporting fuel. The first constraints (2) ensure that all deliveries are satisfied: constant δ_p^i denotes whether or not path p performs delivery i. Constraints (3) ensure that barges sail with tug boats: constant $\delta_{ij\tau}^p$ denotes whether or not path p sails on arc (i, j) at time τ. The number of used barges resp. tug boats is limited in constraints (4), where $|k|$ denotes the number of vehicles of type k. Finally, bounds (5)-(6) force variables to take on feasible values.

Pricing problem for tug boats

Let $\pi_i \in \mathbb{R}$, $\lambda_{ij\tau} \leq 0$ and $\omega_{k_T} \leq 0$ be the dual variables of constraints (2), (3) and (4), respectively. The reduced cost for a tug boat path p is:

$$\bar{c}_p^{k_T} = c_p^{k_T} + \sum_{(i,j)\in A} \sum_{\tau \in T} \lambda_{ij\tau}^{k_T} - \omega_{k_T} \leq 0 \Rightarrow \sum_{(i,j)\in A} \left(c_{ij}^{k_T} + \sum_{\tau \in T} \lambda_{ij\tau}^{k_T} \right) \leq \omega_{k_T} \quad (7)$$

A tug boat must satisfy constraints on time windows and must travel from one depot or power plant to the next such that the reduced cost (7) is minimized. Cycles are allowed, i.e., a path can visit the same depot or power plant several times. The reduced costs can be interpreted as arc weights, hence the pricing problem becomes the *Shortest Path Problem with Resource Constraints* and is solved by a pseudo-polynomial labeling algorithm [1].

Pricing problem for barges

The reduced cost for a barge path p is:

$$\bar{c}_p^{k_B} = -\sum_{i \in L} \pi_i - \sum_{(i,j)\in A} \sum_{\tau \in T} \lambda_{ij\tau} \leq \omega_{k_B} \quad (8)$$

Again, the reduced cost (8) must be minimized according to constraints on time windows and path connectivity. By subtracting π_i from all edges into i, the reduced costs can be viewed as arc weights and a barge path can be seen as pairs of depots and deliveries, where each pair must add to the negative reduced cost. Enumerating all pairs of depots and deliveries gives $\mathcal{O}(|D||L||T|^2)$ pairs. Combining these pairs into a barge route can be done with a dynamic programming approach, and this gives a pseudo-polynomial algorithm for the pricing problem.

Branching

Branching is necessary, when the solution in the current branch-and-bound node is fractional. A solution with binary barge path variables can be transformed into being integer using Proposition 4 in [7], hence branching on barge usage of arcs at specific times eventually ensures a feasible solution and is a finite strategy.

3 Computational Evaluation

The proposed BP-price algorithm is implemented and compared to solving the mathematical formulation using a standard MIP-solver. The solution methods are tested on instances based on real-life data provided by DONG Energy, Denmark. A test instance consists of a number of deliveries at a number of power plants. The start time of a delivery is defined by the time at which the plant has enough capacity to stock a delivery of fuel. The end time of a delivery is defined by the time at which the plant reaches the least allowed amount of available fuel. Time is discretized into hours, i.e., each time stamp represents an hour. The instance covering a full year's fuel consumption consists of 122 deliveries, which all have large time windows.

The proposed BP-algorithm is implemented using the COIN Bcp framework [5]. CPLEX 12.1 is used for solving the MIP formulation. The presented work is still in progress, hence a full computational evaluation has not yet been performed. Early results suggest that the BP-algorithm outperforms solving the original formulation.

Current performance of the BP-algorithm indicates the necessity of a good primal heuristic. Without such a heuristic, the BP-algorithm converges slowly and generates a very large number of columns; the far majority of these columns does not improve the solution. The many columns are caused by large time windows giving rise to many different arrival and departure times. Also, the reduced costs only indicate that expensive arcs should not be used at certain times instead of indicating that the arcs should *never* be used. Finally, ensuring that barges and tug boats sail together adds to the number of columns.

References

[1] J. E. Beasley and N. Christofides. An algorithm for the resource constrained shortest path problem. *Networks*, 19:379–394, 1989.

[2] I.-M. Chao. A tabu search method for the truck and trailer routing problem. *Computers and Operations Research*, 29(1):33–51, 2002.

[3] M. Drexl. *On Some Generalized Routing Problems.* PhD thesis, RWTH Aachen University, 2007.

[4] B. Kallehauge, J. Larsen, O. Madsen, and M. Solomon. Vehicle routing problem with time windows. *Column Generation*, pages 67–98, 2005.

[5] R. Lougee-Heimer. The common optimization interface for operations research. *IBM Journal of Research and Development*, 47:57–66, 2003.

[6] P. Toth and D. Vigo. *The Vehicle Routing Problem.* SIAM - Monographs on Discrete Mathematics and Applications, Philadelphia, 2002.

[7] F. Vanderbeck. On Dantzig-Wolfe decomposition in integer programming and ways to perform branching in a branch-and-price algorithm. *Operations Research*, 48(1):111–128, 2000.