A Non-Hermitian Approach to Non-Linear Switching Dynamics in Coupled Cavity-Waveguide Systems

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A Non-Hermitian Approach to Non-Linear Switching Dynamics in Coupled Cavity-Waveguide Systems

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Abstract: We present a non-Hermitian perturbation theory employing quasi-normal modes to investigate non-linear all-optical switching dynamics in a photonic crystal coupled cavity-waveguide system and compare with finite-difference-time-domain simulations.

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Photonic crystal (PhC) membrane structures has attracted much attention as a platform for integrated all-optical circuits performing signal processing tasks at ultra high bandwidths with low energy consumption in future optical communication systems [1]. Coupled cavity-waveguide systems provide enhanced non-linear light-matter interaction and small footprint, which are essential for efficient optical integration. Such systems may be conveniently described by coupled mode theory (CMT) [2], in which the optical field in the cavity is expanded on the eigenmodes of the cavity. In general, these modes are leaky, corresponding to a finite quality factor of the cavity, and they are solutions to the equation:

$$\nabla \times \nabla \times \tilde{\mathbf{f}}_{\mathrm{C}}(\mathbf{r}) - \frac{\omega_{\mathrm{C}}^2 \varepsilon_{\mathrm{r}}(\mathbf{r})}{c^2} \tilde{\mathbf{f}}_{\mathrm{C}}(\mathbf{r}) = 0, \tag{1}$$

with outgoing wave boundary conditions (the Sommerfeld radiation condition) [3]. The relative permittivity is denoted ε_{r} and c is the speed of light in vacuum. When imposing these boundary conditions, the eigenfunctions are quasi-normal modes and the eigenvalues ω_{C} are complex, so common textbook results based on Hermitian perturbation theory cannot be applied to calculate changes in the eigenvalues due to non-linear interactions. Instead, non-Hermitian perturbation theory [4] must be applied, and in this work, we present the result of this approach in a 2D problem with the Kerr effect as the non-linear interaction. Fig. 1 shows the model PhC structure consisting of a finite sized

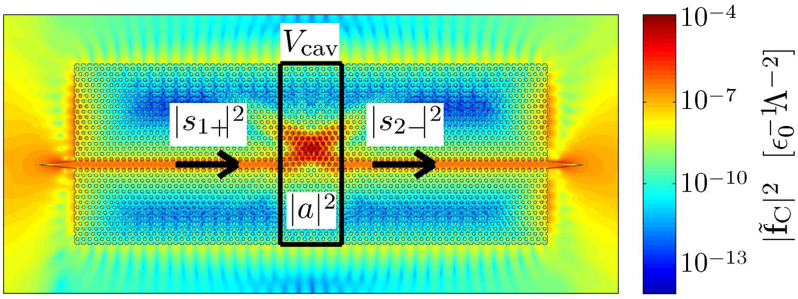

Fig. 1. Field distribution of the quasi-normal mode $|\tilde{\mathbf{f}}_{\mathrm{C}}|^2$ (logarithmic scale). The dielectric structure is also indicated as well as the cavity volume V_{cav} (black rectangle), the incoming power $|s_{1+}|^2$, the outgoing power $|s_{2-}|^2$, and the energy in the cavity $|a|^2$.

membrane with air holes. The figure also depicts the quasi-normal mode $|\tilde{\mathbf{f}}_{\mathrm{C}}|^2$ and the variables of the CMT model that consists of equations governing the time dependence of the fields $a(t)$, $s_{1+}(t)$, and $s_{2-}(t)$:

$$\frac{da(t)}{dt} = -i(\omega_{\mathrm{C}} - \omega_{\mathrm{L}})a(t) + i\omega_{\mathrm{C}} \frac{|a(t)|^2}{U_{11}} a(t) + \sqrt{\frac{2}{\tau_{\mathrm{wg}}}} s_{1+}(t) \quad \text{and} \quad s_{2-}(t) = \sqrt{\frac{2}{\tau_{\mathrm{wg}}}} a(t), \tag{2}$$

where ω_{L} is the carrier frequency of the incoming field and $2/\tau_{\text{wg}} = -\text{Im}\{\omega_{\text{C}}\}$ when the cavity only couples to the waveguides. The complex characteristic energy U_{11} is found from Non-Hermitian perturbation theory and is given by:

$$\frac{1}{U_{11}} = \lim_{V\to\infty}\left[\frac{3}{8}\frac{\int_V \chi^{(3)}(\mathbf{r})|\tilde{\mathbf{f}}_{\text{C}}(\mathbf{r})|^2\tilde{\mathbf{f}}_{\text{C}}(\mathbf{r})\cdot\tilde{\mathbf{f}}_{\text{C}}(\mathbf{r})dV}{\int_V \varepsilon_{\text{r}}(\mathbf{r})\tilde{\mathbf{f}}_{\text{C}}(\mathbf{r})\cdot\tilde{\mathbf{f}}_{\text{C}}(\mathbf{r})dV + i\frac{c\sqrt{\varepsilon_{\text{B}}}}{2\omega_{\text{C}}}\int_{\partial V}\left[\tilde{\mathbf{f}}_{\text{C}}(\mathbf{r})\cdot\tilde{\mathbf{f}}_{\text{C}}(\mathbf{r})\right]\hat{\mathbf{r}}\cdot d\mathbf{A}}\right], \tag{3}$$

where $\chi^{(3)}$ is the third order susceptibility, ε_{B} is the background permittivity, ε_0 is the vacuum permittivity, and μ_0 is the vacuum permeability. Eq. (3) is our main result, and in Fig. 2, we show that it provides a very good agreement between-finite-difference-time-domain (FDTD) calculations and CMT results obtained from Eq. (2). Fig. 2 also indicates the

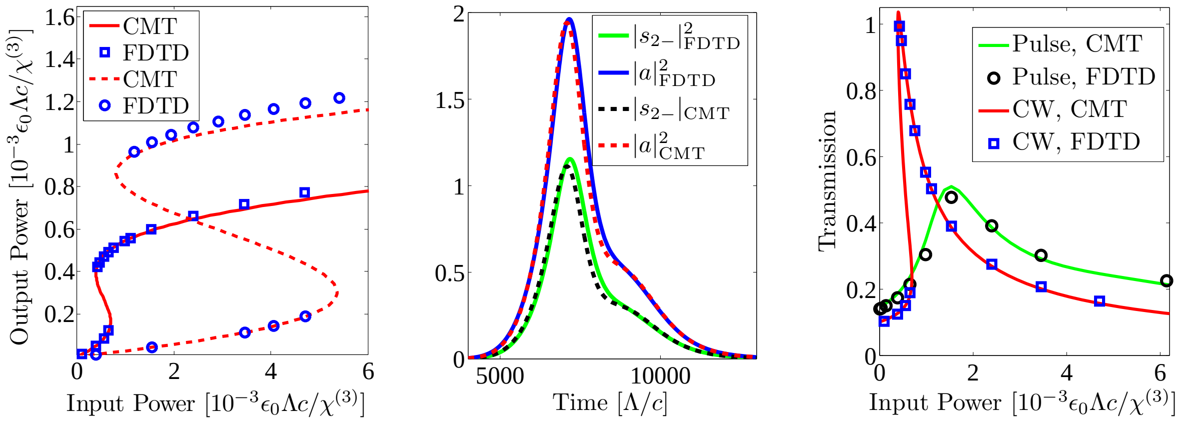

Fig. 2. Left: Output power as a function of input power for a CW input field for two different detunings: $\omega_{\text{C}} - \omega_{\text{L}} = -3\text{Im}\{\omega_{\text{C}}\}$ (solid red and blue squares), and $\omega_{\text{C}} - \omega_{\text{L}} = -6.2\text{Im}\{\omega_{\text{C}}\}$ (dashed red and blue circles). Center: Cavity energy and output power as a function of time for a pulse input field with peak power $3.4\times10^{-3}\varepsilon_0\Lambda c/\chi^{(3)}$ and detuning $\omega_{\text{C}} - \omega_{\text{L}} = -3\text{Im}\{\omega_{\text{C}}\}$. Energy is in units of $[\varepsilon_0\Lambda^2/\chi^{(3)}]$, where Λ is the lattice constant of the PhC. Right: Transmission of pulses (solid green and black circles) and CW input (solid red and blue squares) as a function of input peak power for a detuning of $\omega_{\text{C}} - \omega_{\text{L}} = -3\text{Im}\{\omega_{\text{C}}\}$.

limitations of CMT as the agreement with FDTD calculations becomes worse for large detuning values or large input powers.

CMT is a powerful technique for modeling coupled cavity-waveguide systems, but in a rigorous and quantitative formulation of the theory, the leaky nature of the cavity modes leads to important differences from standard textbook results. Our approach is not limited to Kerr nonlinearities, but can be applied to other types of linear or nonlinear perturbations of Eq. (1). Therefore, we believe this work represents an important progress in the understanding of coupled mode theory and its applicability in modeling non-linear photonic crystal cavity-waveguide systems.

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