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Metamaterial composite bandpass filter with an ultra-broadband rejection bandwidth of up to 240 terahertz

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We present a metamaterial, consisting of a cross structure and a metal mesh filter, that forms a composite with greater functional bandwidth than any terahertz (THz) metamaterial to date. Metamaterials traditionally have a narrow usable bandwidth that is much smaller than common THz sources, such as photoconductive antennas and difference frequency generation. The composite structure shown here expands the usable bandwidth to exceed that of current THz sources. To highlight the applicability of this combination, we demonstrate a series of bandpass filters with only a single pass band, with a central frequency \((f_0)\) that is scalable from 0.86–8.51 THz, that highly extinguishes other frequencies up to \(>240\) THz. The performance of these filters is demonstrated in experiment, using both air biased coherent detection and a Fourier transform infrared spectrometer (FTIR), as well as in simulation. We present equations—and discuss their scaling laws—which detail the \(f_0\) and full width at half max (\(\Delta f\)) of the pass band, as well as the required geometric dimensions for their fabrication using standard UV photolithography and easily achievable fabrication linewidths. With these equations, the geometric parameters and \(\Delta f\) for a desired frequency can be quickly calculated. Using these bandpass filters as a proof of principle, we believe that this metamaterial composite provides the key for ultra-broadband metamaterial design. © 2014 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4875795]

Since their introduction nearly fifteen years ago, metamaterials1–3 have provided a unique design paradigm. The concept of individually tuning the permittivity and permeability—or index and impedance, if you prefer—excited the community with a negative index of refraction,2,3 imaging past the diffraction limit,4 and transformation optics.5 Those lofty motivations have gradually devolved into more traditional, but imminently practical, applications such as dielectric sensing,6 polarization control,7,8 modulation,9 absorption,10 and detection.11,12 However, the initial motivation of designing the permittivity and permeability often limits the study of metamaterials to a small bandwidth around their design frequency. This bandwidth limitation is due to several factors such as the resonant nature of most metamaterials, the appearance of unwanted higher order modes, and the prevalence of the effective medium approximation where metamaterials are only considered to have a “designed” permittivity and permeability in the deep sub-wavelength regime.13 While this behavior can be easily integrated with narrowband applications, the two most common methods of terahertz (THz) generation, photoconductive antennas,14 and nonlinear generation in either crystals15 or air plasma,16 both yield broad spectra that suggest there is an unfulfilled need for broadband THz components.

Here, we reconcile the disconnect between traditional broadband sources and narrowband metamaterial components by demonstrating that a metamaterial composite can have an ultra-broadband usable range. As a proof of principle of this ultra-broadband concept, we have made a series of bandpass filters that display the largest usable bandwidth of any THz metamaterial device to date. The filters have a single pass band, with a central frequency scalable between 0.86–8.51 THz, while severely attenuating all other frequency components to \(>240\) THz. These filters, which operate due to a trapped mode excitation, were originally introduced to the THz regime by Paul et al., from 0–2.5 THz.17 We have customized their structure and expanded the bandwidth by almost two orders of magnitude.

To clearly identify the two constituent components in the metamaterial composite, a cross element, and a metal mesh,18,19 we present an optical picture of a 2 \(\times\) 2 array of unit cells in Figure 1(a). The unit cell in the bottom right, outlined in black, clearly identifies the cross component. However, this unit cell choice suggests that the cross is placed inside of its own, slightly larger, Babinet complement.20 If the unit cell is translated by \((-1/2, 1/2) \times P\), to the grey outlined unit cell, a different structure is suggested. In this new unit cell, if the cross is ignored, it can be seen that the Babinet complement is also a metal mesh filter.

The sample dimensions in the figure—for except for \(e\), which was held constant at 1.5 \(\mu m\)—are all scaled by a single dimensionless scaling parameter \((\sigma)\) according to the following equations: \(L = 7.09 \times \sigma\); \(W = 4.5 \times \sigma\); \(P = 10.03 \times \sigma\), where all dimensions are in microns. The cross element will always have a length and width of \(L = -2 \times e\) and \(W = 2 \times e\), respectively. The constant value of \(e\) provides an easily achievable minimum linewidth for fabrication using UV photolithography, and the samples are polarization insensitive due to their four fold symmetry. We fabricated two different styles of samples: single- and double-sided. Both styles have the exact same metamaterial pattern, except that the double-sided
The structure has the pattern on both sides of the 525 μm thick high resistivity silicon (HR-Si) substrate as shown in Fig. 1(b). The dimensions, σ values, central frequency (f₀), and full width at half max (Δf) of the filters studied are presented in Table I. For reference, these dimensions were originally chosen so that the higher order modes of the cross geometry would couple to lattice modes to eliminate unwanted transmission modes, although this ended up being unnecessary as discussed later.

To highlight the effect of the two constituent components in the structure, we present the simulated transmission curves of the cross, metal mesh, and full composite structure in Figure 2. The metal mesh is easily described as a combination of inductive and capacitive meshes. A capacitive mesh, which is simply a two dimensional array of metallic squares, works as a low pass filter. The complementary structure, called an inductive mesh, is a wire grid that functions as a high pass filter. Their combination, a metal mesh filter as shown here, has a pass band roughly following the transmission of the HR-Si wafer. The second difference between the curves is the appearance of sharp modes on the transmission of the full structure. These lines can all be attributed to lattice modes, and are calculated using

$$ f_j = \frac{c_n \sqrt{f^2 + k^2}}{P} $$

and all frequencies are in THz.

![Image of fabricated samples](image_url)

**FIG. 1.** Optical pictures of fabricated samples. (a) A 2 x 2 picture of the composite filter. The two different choices of unit cells, represented by the black and grey outlines, help identify the cross and metal mesh, respectively. The minimum linewidth (ε) is held constant at 1.5 μm for every sample, L, W, and P all scale with the dimensionless parameter, σ, as described in the text and Table I. The layer is entirely gold except for the small linewidth defined by ε, which is HR-Si. (b) A single-sided sample (right) and a double-sided sample (left). The mirror clearly shows the double-sided sample’s second metallization layer on the backside of the HR-Si substrate. The cross section of the single-sided sample is gold/HR-Si, while the double-sided sample is gold/HR-Si/gold.

<table>
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<th>L</th>
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<th>P</th>
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<th>Δf₁</th>
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**TABLE I.** The physical dimensions of the fabricated samples, their simulated central frequency (f₀), and full width half max for single- (Δf₁) and double-sided (Δf₂) styles. σ is dimensionless, L, W, and P are in microns, and all frequencies are in THz.

**FIG. 2.** The simulated transmission spectrum of a double-sided metamaterial and metal mesh filter composite. The dimensions correspond to the σ = 2 structure in Table I. The red line is the transmission of the cross element without the metal mesh, the blue line is the metal mesh filter without the cross, the black line is the full metamaterial composite, and the dashed magenta line is a single element instead of a full periodic array. The dotted lines represent the geometric ratio of metallization area divided by unit cell area times the Fresnel transmission coefficient of the HR-Si wafer.
modes of the cross element \((n = 3 \text{ and } n = 5, \text{ respectively})\), but their highly oscillatory current distribution likely explains their weak coupling to the incident plane wave, as well as the absence of any other transmission bands.

To verify these simulated results, the fabricated samples were measured with THz-time domain spectroscopy using a two-color air plasma for generation\(^{23}\) and air biased coherent detection.\(^{24}\) This system does have a non-traditional Bessel-Gauss beam profile,\(^{25}\) but this beam profile is simply a superposition of Gaussian beams and is subsequently irrelevant to the filter performance. The optical pulse length used to generate the air plasma and THz beam was 35 fs, yielding an anticipated bandwidth of \(\sim 1/35 \text{ fs} = 28.6 \text{ THz}\) which is approximately that achieved in our reference measurements. Further details of the experimental system can be found elsewhere.\(^{26}\) The samples were also measured in a Fourier transform infrared spectrometer (FTIR) to examine their high frequency extinction up to 240 THz (1.25 \(\mu\)m/0.99 eV).

An aggregate comparison between simulation and experiment for all samples is shown in Figure 3. The two plots compare \(f_0\) and \(\Delta f\) versus \(\sigma\) for both the single- and double-sided samples. Fits to the data were conducted using a power law and the simulated values in Table I. The resulting equations are \(f_0 = 8.22 \times \sigma^{-1.42} + 0.28; \Delta f_1 = 5.51 \times \sigma^{-1.56} + 0.22; \) and \(\Delta f_2 = 3.16 \times \sigma^{-1.05} - 0.21,\) where \(\Delta f_1\) is for the single-sided sample and \(\Delta f_2\) is for the double-sided sample. For fit details, see Ref. 27. Note that both the single- and double-sided samples share the same resonance frequency, because, due to the relative thickness of the HR-Si substrate, there is no coupling between these two layers and they can be treated independently at the band pass frequencies.\(^{28}\) The double-sided structures show a reduced \(\Delta f\) due to transmission through two filters, demonstrating that multiple filters can be stacked to achieve an even narrower bandwidth, as required. It is our hope that these design equations can be used to quickly fabricate bandpass filters for any frequency in this range. Simply calculate \(\sigma\) for the desired \(f_0\), use the geometric equations to determine \(L, W,\) and \(P,\) and then calculate \(\Delta f_1\) and \(\Delta f_2\) for the subsequent filters. Since \(L, W,\) and \(P\) are linear with \(\sigma,\) they also have a nonlinear relationship with frequency and the same scaling behavior as \(\sigma.\) They decrease monotonically from \(L = 0.20 \times \lambda_0, W = 0.13 \times \lambda_0,\) and \(P = 0.28 \times \lambda_0\) for \(\sigma = 1\) to \(L = 0.13 \times \lambda_0, W = 0.08 \times \lambda_0,\) and \(P = 0.18 \times \lambda_0\) for \(\sigma = 6.25,\) where \(\lambda_0\) is the free space wavelength at \(f_0.\)

Metamaterials are well-known to be scale invariant, yet our scaling equations are clearly not linear with sample size. This scale invariance is broken by the constant value of \(\varepsilon,\) which results in increased coupling in the trapped mode excitation with increasing \(\sigma\) and causes a red shift in \(f_0.\) As a visual aid, Figure 3(a) has a line that is \(f_0\) of the \(\sigma = 1\) filter scaled linearly with \(\sigma.\) The deviation of the results from this line demonstrates the aforementioned red shift vs \(\sigma.\)

We can model this increased coupling by assuming that the capacitance of the metamaterial composite is dominated by the capacitive coupling between the cross element and the metal mesh filter. We begin by describing the filter as a resonant element, where \(f_0 \sim 1/\sqrt{LC}\) and \(L\) and \(C\) are the total inductance and capacitance of the metamaterial, respectively. Next, we assume that the capacitive coupling between the cross element and the metal mesh filter can be approximated as a parallel plate capacitor with capacitance \(C \sim \text{area/ distance,}\) and this contribution dominates the total capacitance of the structure. Making this substitution for \(C,\) we see that \(f_0 \sim N\sqrt{\varepsilon},\) where \(\varepsilon\) is the distance between the cross element and the metal filter, and every other dependency has been lumped into the unknown variable \(N.\) The scale invariance of Maxwell’s equations tell us that if we assume \(\varepsilon = 1.5 \times \sigma,\) then every dimension would scale linearly and the resonance frequency would match the linear approximation plotted in Figure 3(a). This means that \(f_0 \sim N\sqrt{1.5 \times \sigma \sim \sigma^{3/2}},\) and therefore, \(N \sim \sigma^{-3/2}.\) If we instead hold \(\varepsilon\) constant, as we do in our metamaterial samples, we have \(f_0 \sim N\sqrt{\varepsilon} \sim N \sim \sigma^{-3/2}.\) This yields an exponent of \(-1.5,\) which agrees closely with our fitted value of \(-1.42.\)

The scaling of \(\Delta f\) can be described in a similar manner. If we assume that the majority of the energy stored in the filter is in the electric field of the previously mentioned “capacitor,” we can use \(Q \sim 1/f_0RC\) for a capacitive element. Again using our substitutions for \(f_0\) and \(C,\) we have \(Q \sim 1/f_0\sqrt{C} \sim 1/\sqrt{C} \sim 1/\sqrt{\varepsilon}\) which is constant. Since we also know that \(Q \sim f_0/\Delta f,\) a constant \(Q\) implies that \(f_0\) and \(\Delta f\) scale identically, and, therefore, \(\Delta f_1 \sim \sigma^{-3/2},\) which is again close to the fitted value of \(-1.56.\) While this simple argument ignores any changes due to fringing fields, surface capacitance,\(^{29}\) and inductance, the agreement with the fitted scaling equations suggests that this simple capacitive coupling argument captures the essence of the physics at play.

We have also examined \(\Delta f\) for transmission through up to six filters and have, for the sake of design convenience,
In conclusion, we have shown that a metamaterial composite can have an ultra-broadband usable bandwidth that is suitable for virtually any THz source. We have constructed a series of bandpass filters that clearly demonstrate this concept, provided simple equations that can be used to construct filters at any frequency from 0.86–8.51 THz without need for
simulation or design, and described the nature of the scaling laws in the equations. These filters may be fabricated on both sides of the HR-Si substrate for further bandwidth reduction, and multiple filters may be used to narrow the transmitted spectrum even further as required. It is our hope that this work will bring an easy to fabricate, functional THz component to the laboratory, and expand the reach of meta-material based THz components towards broadband functional components.

We acknowledge financial support from the Danish Council for Independent Research (FTP Projects HI-TERA and THz-COW) and the Carlsberg Foundation.

21CST, Microwave Studio, 2013.
27The fits used the simulated data from Table I and the Matlab Curvefitting Toolbox using a power law $f = A r^p + C$. We have rounded to two decimal points in the text to match the precision of the values in Table I. The full results for $f_0$ are $A = 8.223 (8.168, 8.278)$, $B = -1.424 (-1.449, -1.399)$, $C = 0.2774 (0.2191, 0.3357)$; $\Delta f_1$ are $A = 5.511 (5.376, 5.645)$; $B = -1.557 (-1.661, -1.454)$; $C = 0.2178 (0.07851, 0.3571)$; and $\Delta f_2$ are $A = 3.161 (2.972, 3.35)$; $B = -1.046 (-1.189, -0.9027)$; $C = -0.2098 (-0.4184, -0.001059)$, where the parantheses represent the 95% confidence bounds.