



## Optimization in Nonlinear Structural Dynamics with Reduced Order Models

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## Introduction

### Why account for nonlinear vibration?

- Coupled Nonlinear Dynamics/Aeroelasticity of very Flexible Aircraft
- Vibration-based MicroElectroMechanical Systems (MEMS)
- Long, Light and Flexible Blade of Wind Turbine
- High Speed Compliant Actuator
- Squeal of the Brake System

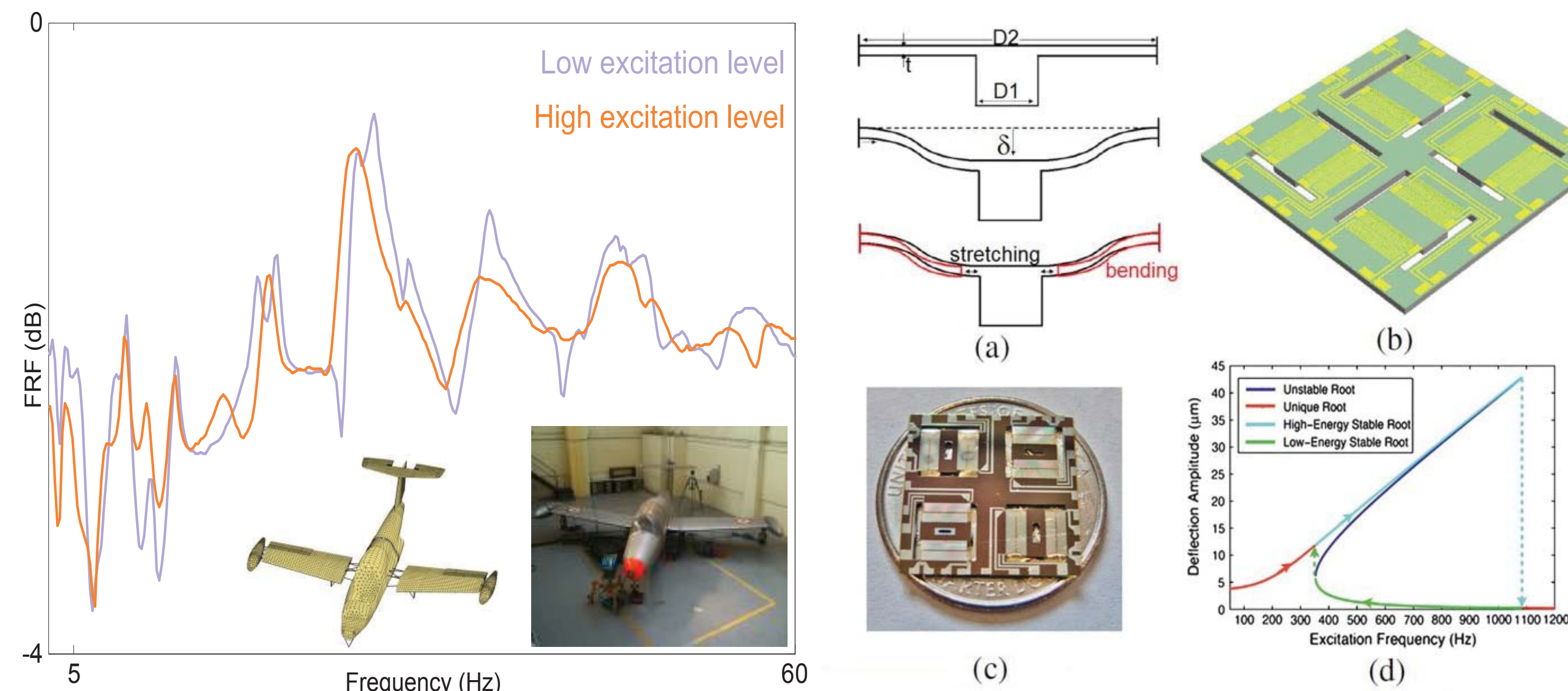


Figure 1: Nonlinear vibration in airplane and MEMS. Left: dynamic testing of an airplane presented by Gaëtan Kerschen [1]; Right: ultra-wide bandwidth piezoelectric energy harvesting device developed by Arman Hajati and Sang-Gook Kim [2].

### What is the problem in optimization?

- Today's design procedures are often based on linear finite element (FE) models.
- Nonlinear structural dynamics is analyzed after a full optimization procedure.
- High computation costs of nonlinear structural dynamics are prohibitive for iterative optimization.

### The goal of this PhD project

Focus on developing reduced-order models to facilitate efficient analysis and optimization.

- Eliminate the time dimension to compute the steady-state vibration efficiently.
- Reduce the spatial dimension to obtain a model with fewer degree-of-freedom.
- Do sensitivity analysis and design optimization using reduced-order models.

## Method

### Time-reduced models

For the time-reduced model, we consider only problems with time-harmonic excitation. These are of major relevance in machinery with rotating parts. The FE model becomes

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{g}(\mathbf{u}) = \mathbf{f}\cos\Omega t$$

in which  $\mathbf{u}$ ,  $\dot{\mathbf{u}}$  and  $\ddot{\mathbf{u}}$  is the discretized displacement, velocity and acceleration vector, respectively. The matrices  $\mathbf{M}$  and  $\mathbf{C}$  represent mass and damping, and  $\mathbf{g}(\mathbf{u})$  is a vector with the nonlinear forces, and  $\mathbf{f}\cos(\Omega t)$  is the time harmonic load.

A semi-analytical method called Incremental Harmonic Balance (IHB) method is used to solve the equation of motion [3, 4]. For the incremental harmonic balance method, the governing equation becomes

$$(\omega^2\bar{\mathbf{M}} + \omega\bar{\mathbf{C}} + \bar{\mathbf{K}}_T(\bar{\mathbf{u}}))\Delta\bar{\mathbf{u}} = \bar{\mathbf{f}} - (\omega^2\bar{\mathbf{M}}\bar{\mathbf{u}} + \omega\bar{\mathbf{C}}\bar{\mathbf{u}} + \bar{\mathbf{g}}(\bar{\mathbf{u}}))$$

in which  $\bar{\mathbf{u}}$  is a vector containing all coefficients of harmonics in Fourier series of  $\mathbf{u}$ . And  $\bar{\mathbf{M}}$ ,  $\bar{\mathbf{C}}$  and  $\bar{\mathbf{K}}_T$  are augmented matrices of mass, damping and tangent stiffness, respectively. And  $\bar{\mathbf{g}}$  and  $\bar{\mathbf{f}}$  are augmented vectors of elastic force and external force, respectively.

### Space-reduced models (future work)

In case of transient loads, e.g. encountered in machinery start-up or with impact loads such as blasts, we need to consider the time response of the structure. The finite element model in this case becomes

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{g}(\mathbf{u}) = \mathbf{f}(t)$$

in which  $\mathbf{f}(t)$  is the specific time-dependent load. Instead of computing the transient response for the full FE model, an analysis model based on nonlinear modal reduction will be applied. The space-reduced model becomes

$$\mathbf{M}_r\ddot{\mathbf{q}} + \mathbf{C}_r\dot{\mathbf{q}} + \mathbf{g}_r(\mathbf{q}) = \mathbf{f}_r(t)$$

in which  $\mathbf{u}$  has been reduced to a set of generalized coordinates  $\mathbf{q}$ . And  $\mathbf{M}_r$  and  $\mathbf{C}_r$  are reduced matrices for mass and damping, respectively. And  $\mathbf{g}_r$  and  $\mathbf{f}_r$  are reduced vectors for elastic force and external force, respectively. The use of nonlinear modal reduction can potentially reduce computational costs by orders of magnitude.

### Design optimization using reduced-order models

A general optimization problem concerning vibration is to minimize the amplitude of vibration. Based on the time-reduced models, this problem can be expressed as

$$\begin{aligned} \min_{\rho_e} c(\rho_e, \omega(\rho_e)) &= \bar{\mathbf{u}}^T \mathbf{L} \bar{\mathbf{u}} \\ \text{s.t. : } \omega^2 \bar{\mathbf{M}} \bar{\mathbf{u}} + \omega \bar{\mathbf{C}} \bar{\mathbf{u}} + \bar{\mathbf{g}} &= \bar{\mathbf{f}}, \\ \sum_{e=1}^{N_e} \rho_e V_e - V^* &\leq 0, V_e = L_e A_e, \\ 0 < \rho_{min} \leq \rho_e \leq 1, V^* &= \alpha V_0, \\ A_e &= \rho_e A_{max}, V_0 = L_e A_{max}. \end{aligned}$$

where  $\rho_e$  are design variables,  $N$  denotes the total number of design variables, the symbol  $\alpha$  defines the volume fraction,  $V_0$  is the volume of the admissible design domain, and  $V^*$  is the given available volume of solid material.

## Examples

### Design of nonlinear beam dynamics

The structure is a doubly clamped beam with periodic load applied at the midspan. The design variable is the width  $w(x)$ . The objective function will be given in each example. The nonlinearity in the model arises from the midplane stretching. The axial strain  $\epsilon_0$  and the curvature  $\kappa$  are defined as

$$\epsilon_0 = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2, \kappa = \frac{\partial^2 w}{\partial x^2}$$

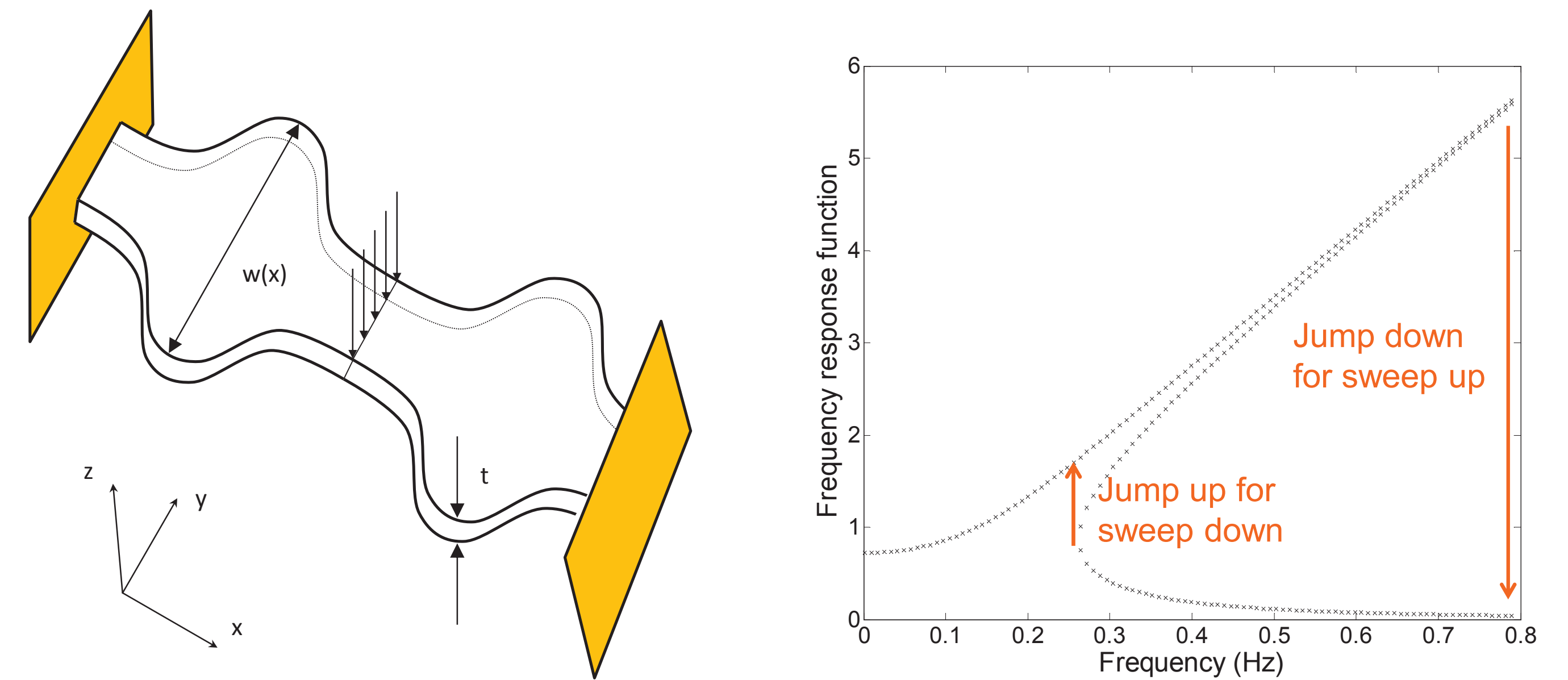


Figure 2: Schematic of the model and a typical frequency-amplitude curve.

### Minimize the resonant peak

$$\min_{\rho_e} c(\rho_e, \omega(\rho_e)) = \bar{\mathbf{u}}^T \mathbf{L} \bar{\mathbf{u}} = a_{11}^2 + b_{11}^2$$

in which  $a_{11}$  and  $b_{11}$  are the coefficients of the fundamental harmonic  $a_{11} \cos(\omega t) + b_{11} \sin(\omega t)$  for the deflection at the midspan.

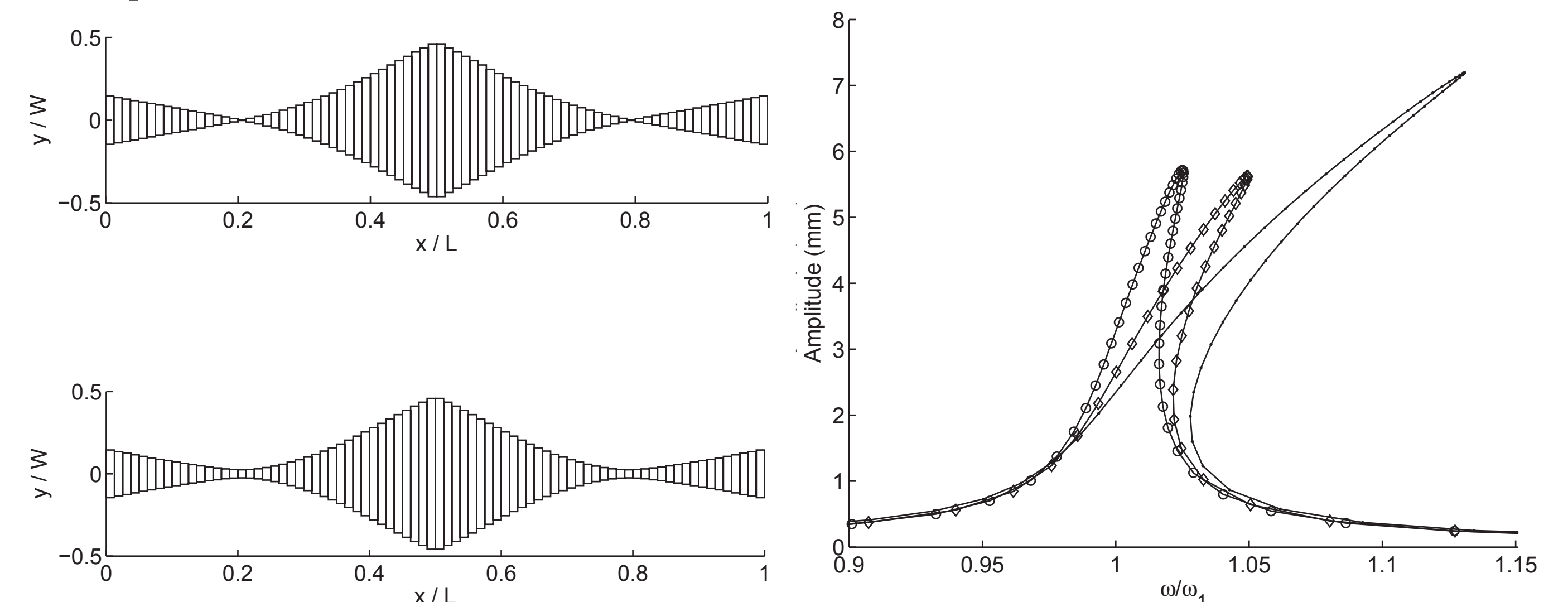


Figure 3: Optimized width for minimizing the resonant peak using linear FE model (left top) and using nonlinear FE model (left bottom), and nonlinear frequency-amplitude curves: circles denote optimized width using linear FE model, diamonds denote optimized width using nonlinear FE model and dots denote uniform width.

### Maximize the super-harmonic resonance

$$\max_{\rho_e} c(\rho_e, \omega(\rho_e)) = \bar{\mathbf{u}}^T \mathbf{L} \bar{\mathbf{u}} = a_{13}^2 + b_{13}^2$$

in which  $a_{13}$  and  $b_{13}$  are the coefficients of the third-order harmonic  $a_{13} \cos(3\omega t) + b_{13} \sin(3\omega t)$  for the deflection at the midspan.

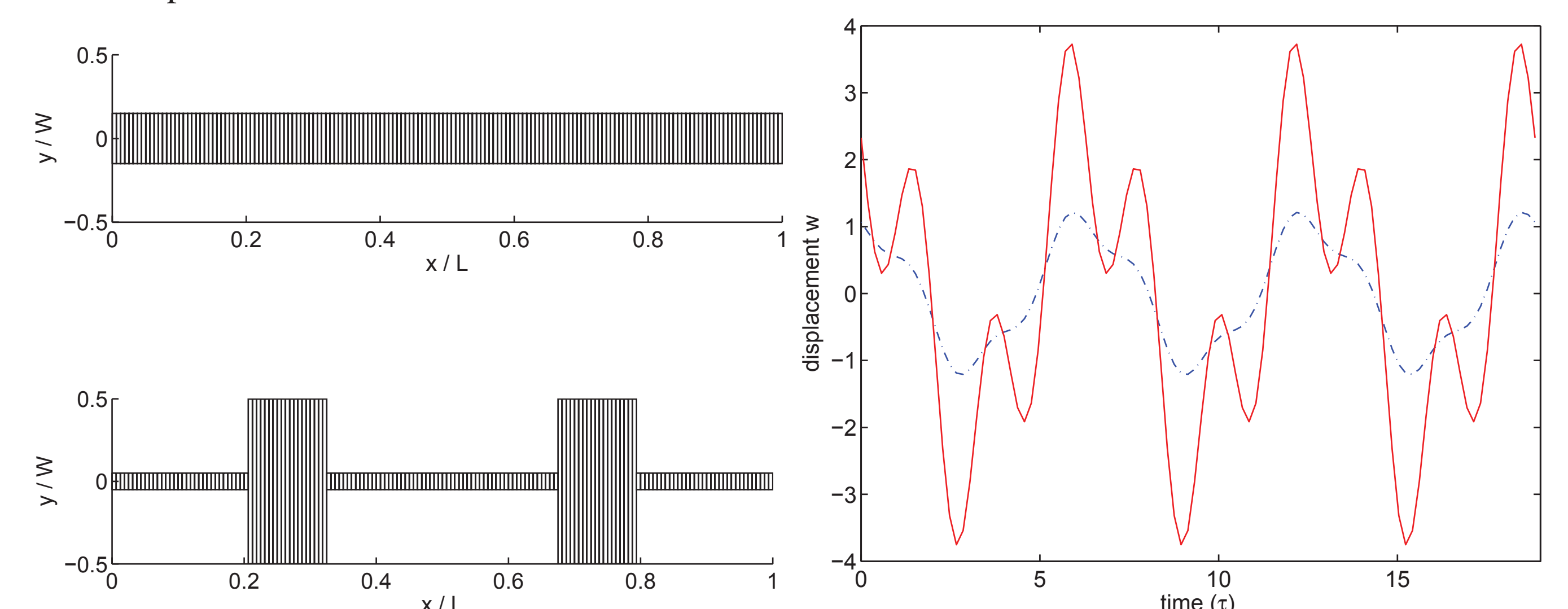


Figure 4: Optimized width for maximizing super-harmonic resonance (Left top: uniform width; Left bottom: optimized width) and the responses before optimization (dashed line) and after optimization (solid line).

## Discussion

- Optimized width for minimizing the resonant peak using nonlinear FE model does not have "weak" links.
- Nonlinear structural dynamics is essential for optimizing high-order harmonics in the response.

## References

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