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*Publication date:*
2014

[th] U.S. National Congress on Theoretical and Applied Mechanics
Michigan State University, June 15-20, 2014

**Abstract: D-02-507**

# Analytical sensitivity analysis and topology optimization of nonlinear resonant structures with hardening and softening behavior

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With the increasing applications of nonlinear resonant structures [1], researchers are interested in controlling the hardening and softening behavior [2]. We present a systematic procedure for topology optimization of nonlinear resonant structures with hardening and softening behavior. The combination of the finite element method (FEM) and alternating frequency/time domain method (AFT) is used to compute the nonlinear vibrational response. The finite element model facilitates an element-density-based parameterization for topology optimization [3] and the alternating frequency/time domain method takes advantage of the ease in evaluating complex nonlinearities in time domain [4].

Mechanical structures that possess geometrical nonlinearities are used to demonstrate the applicability and efficiency of the proposed procedure, and is here illustrated using 2D continuum structures. Two kinds of optimization problems are considered, including bandwidth problems and dynamic compliance problems. For bandwidth problems, the relative and absolute frequency shifts of resonance peaks caused by hardening and softening behavior are optimized. For dynamic compliance problems we consider point vibration amplitudes and total energy levels of the structure as optimization objectives. In order to speed up the computations we apply analytical sensitivity analysis [5] instead of the commonly applied finite difference approximation. For large FE models this leads to computational savings on up to several orders of magnitude. The optimization problem for relative frequency shift can be formulated as:

$$\begin{aligned} &\min_{x_e} \ \pm\gamma \\ \text{s.t. :}\ &\gamma = \frac{\omega^* - \omega_L}{\omega_L}, \quad \mathbf{K}_L\Phi = \omega_L^2\mathbf{M}\Phi, \quad (\omega^*)^2\bar{\mathbf{M}}\bar{\mathbf{q}} + (\omega^*)\bar{\mathbf{C}}\bar{\mathbf{q}} + \bar{\mathbf{g}} = \bar{\mathbf{f}}, \quad b_{i1} = 0, \\ &E_e = E_{\min} + (x_e)^p(E - E_{\min}), \quad \rho_e = \rho_{\min} + (x_e)^q(\rho - \rho_{\min}), \\ &\sum_{e=1}^{N_e} x_e \le \alpha N_e, \quad 0 \le x_e \le 1. \end{aligned} \tag{1}$$

where $\omega^*$ is the nonlinear resonant frequency at the specified load, $\omega_L$ is the linear resonant frequency, $\gamma$ measures the extent of hardening and softening behavior, $E_e$ and $\rho_e$ are interpolated Young's modulus and mass density of each element, $p$ and $q$ are integers with typical values 3 and 1, $x_e (e = 1, \ldots, N_e)$ are design variables and $\alpha$ represents a maximum allowable volume fraction. Note that $b_{i1}$ is the coefficient of one sinusoidal term and the condition $b_{i1} = 0$ is related to the phase lag quadrature criterion [6]. The barred matrices and vectors are in the frequency domain and can be found in [5].

**Example** Consider a thin 2D continuum structure in micro-scale with doubly-clamped boundary condition. An in-plane vertical time-harmonic load is applied at the center node of the structure. The material is Silicon nitride and its mechanical properties are: Young's modulus $E = 241$ GPa, Poisson's ratio $\nu = 0.23$ and mass density 300 kg/m$^3$. Plane stress conditions are assumed and preliminary results are obtained with a coarse discretization of $40 \times 20$ elements. The initial design is simply the uniform distribution of material in the design domain. The specified load is so small that the relative frequency shift around the first flexural mode is less than 0.04% for the initial design. The objective function is to maximize the relative frequency shift caused by hardening resonance around the first flexural mode. In the optimized design, the solid structure (black area) is not allowed to fill more than half of the total area of the design domain. The second optimized design is obtained by imposing an additional constraint on minimal eigenvalue in order to ensure sufficient structural rigidity.

For both optimized designs shown in Figure 1, the relative frequency shifts are more than 34%. A physical interpretation of the optimized structures is that the midplane stretching plays a key role when four beams are connected to one central mass and two boundaries through eight hinge-like connections. In the designs point-hinges appear as well as gray elements that correspond to intermediate material properties. Further work is currently being undertaken to improve the designs by employing a finer mesh, projection filtering and minimal length control techniques. However, in reality the gray elements may be realized using a weaker material such as polymer. For example, polymeric hinges have been used in the design of micro-mirrors to achieve large deflection at small external forces.

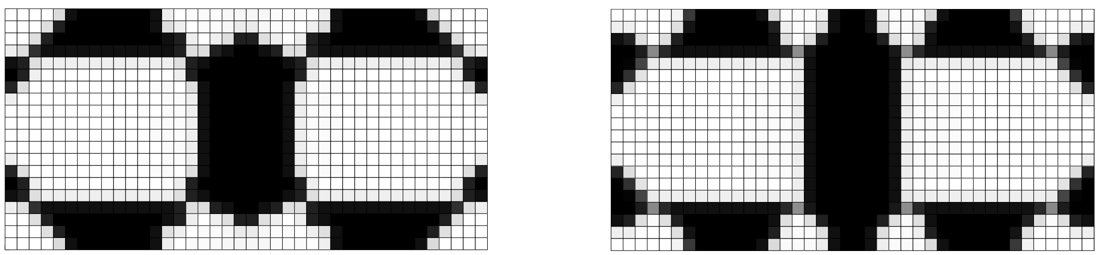

**Figure 1.** Optimized design with constraint on maximal volume (left) and an additional constraint on minimal eigenvalue (right)

**Conclusion** A systematic procedure is proposed for optimization of nonlinear resonant structures with hardening and softening behavior by tailoring the geometric nonlinearity. The combination of analytical sensitivity analysis and the topology optimization procedure is shown to be a promising tool in design of nonlinear micro- and nano-resonators.

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