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UPPER BOUND ANALYSIS OF EQUAL CHANNEL ANGULAR EXTRUSION

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Summary: The relative pressure and total shear at equal-channel angular extrusion (ECAE) in a nonrectangular die were determined by upper bound analysis and discrete velocity field. The obtained data were compared with the slip line solution of V. Segal. Good agreement between the two results was found. Physical modeling by plasticine confirmed the appearance of a dead zone at ECAE. The development of dead zone was analytically investigated.

Keywords: Equal Channel Angular Extrusion, Upper-Bound Analysis, Physical Modeling

INTRODUCTION

Equal Channel Angular Extrusion (ECAE), also known as Equal Channel Angular Pressing (ECAP), involves one or several passes of extrusion of a lubricated billet in a die with two intersecting channels of equal cross-section (Fig. 1a). This process has been very promising for production of bulk ultra fine-grained materials with special properties. The large cumulative shear strain at ECAE leads to refinement of grains and to increase of their boundary length. The obtained materials show the combination of a very high strength and ductility [1].

Figure 1: a – ECAE layout: 1 and 4 - intersecting channels, 2 - punch and 3 - sample; b – rigid blocks partitioning scheme; c – velocity hodograph

For an analytical investigation of ECAE pressure and resulting shear Segal V. M. applied the slip line approach [2,3]. Beyerlein I. J. and Tomé C. N. used a kinematic analysis of strains during ECAE [4]. Alkorta J. et al. [5], Altién B. S. et al. [6], Laptev A. M. et al. [7,8] and Eivani A. R. et al. [9] studied ECAE by upper bound
anal\textsuperscript{ysis}. For visualization of metal flow during ECAE and indication of possible problems at real process Manna R. et al. carried out a physical modeling of this process \cite{10}. In the present work we combine the physical modeling and theoretical analysis of ECAE by upper bound theory and discrete velocity field. The development of a dead zone was investigated by optimization of analytical solution.

2. PHYSICAL AND MATHEMATICAL SIMULATION

2.1. Plasticine modeling

First of all the material flow at ECAE was studied using plasticine. The rectangular plasticine billet with cross-section of 30x20 mm\textsuperscript{2} was extruded in the wood die as it is shown in Fig. 2. The die had two equal channels with intersection angle of 2\(\theta = 105^\circ\). The material flow was recorded by camera through the transparent wall of the die made from Plexiglas. The talcum powder was used as a lubricant. The appearance of the dead zone near the external die corner was observed. The dead zone was symmetrical at the beginning of ECAE. The dead zone retained its length in the entrance channel during process development, but its length in outlet channel became larger. As a result the dead zone transformed to asymmetrical one. The gap between the plasticine billet and the die wall was also found in the outlet channel.

2.2. Upper bound analysis

According to the upper bound approach a trial velocity field has to be introduced. The velocity field can be continuous, discontinuous or mixed. In the present work the discontinuous velocity field was used. The 2D plane model of sample was divided into 4 rigid triangular sections, as shown in Fig. 1b. The appearance of a symmetrical dead metal zone was assumed because its asymmetry leads to violation of material incompressibility at used rigid blocks division. This assumption is in contrast to results of plasticine modeling, but it can be used as a first approximation. Opposite the more complex blocks division should be tried. The length of symmetrical dead zone was characterized by \(h = a \cdot x\). Here \(a\) is the channel width; \(x \in [0,1.0]\) is the relative length of the dead zone in both entrance and outlet channels. The friction only along the lines AC and DB was taken into account by Tresca friction law. Corresponding velocity hodograph is shown in Fig. 1c. Further the ECAE pressure on the line AO was calculated. The extruded material was discussed as rigid-plastic with no strain-hardening. The friction forces were assumed as independent on sliding velocity. The balance of external and internal powers at plastic deformation was expressed by equation

\[
p \cdot a \cdot V_i = k \cdot (l_{i-2} V_{i-2} + l_{i-3} V_{i-3} + l_{1-4} V_{1-4}) + 2mk \cdot (l_{AC} + l_{DB}) \cdot V_1,
\]

where \(p\) is a applied pressure; \(V_1\) is a velocity in entrance and output channels; \(k\) is a shear strength of extruded material; \(m \in [0;0.5]\) is a friction factor in Tresca law expressed by equation \(\tau_f = 2mk\); \(l_{i-j}\) is the length of join interface of blocks \(i\) and \(j\); \(V_{i-j}\) is a relative sliding velocity of these blocks. The terms in equation (1) were expressed as functions of velocity \(V_1\) and relative length of the dead zone \(x = h/a\). After substitution of obtained relationships in equation (1) and algebraic transformation, the formula for calculation of relative pressure was derived

\[
p \cdot \frac{a \cdot V_1}{2k} = 1 + x \cdot \tg \theta + (\ctg \theta - x)^2 + 2m(\ctg \theta - x),
\]

where \(2\theta\) is an intersection angle between entrance and outlet channels (Fig. 1a).
Equal channel angular extrusion is a technique for grain refinement. Therefore the estimation of resulting plastic ECAE shear is also important. The total ECAE shear $\gamma$ is the sum of the shears on the discontinuity lines $CO$ and $DO$ in Fig. 1b, i.e.

$$\gamma = \gamma_{1-2} + \gamma_{2-3}.$$  

(3)

It is known that

$$\gamma_{i-j} = [V_{i-j}]t_{i-j},$$

(4)

where $V_{i-j}$ is a velocity component orthogonal to a discontinuity line $l_{ij}$ [11]. Using hodograph in Fig. 1c the following relationship was obtained

$$\gamma = \frac{2(1 + (\text{ctg} \theta - x)^2)}{\text{ctg} \theta + (\text{ctg} \theta - x)}.$$  

(5)

According to the upper bound theory the best approximation of the real $p/2k$ and $\gamma$ corresponds to the minimum of expression (2). The dependence of $p/2k$ on relative length of dead zone $x$, friction factor $m$ and angle $2\theta$ was numerically investigated. The corresponding diagrams are presented in Fig. 3.

![Figure 3: Relative ECAE pressure p/2k versus x and m at different channels intersection angle](image)

Analysis of (2) shows that minimum of this function takes place when

$$x = \left[\frac{1 + 2m}{1 + \text{tg}^2 \theta} - \text{tg} \theta \cdot \sqrt{2 \cdot (1 + 2m) \cdot (1 + \text{tg}^2 \theta)}}\right]\left[(1 + 2m) \cdot \text{tg} \theta\right].$$  

(6)

In same cases the global minimum of $p/2k$ is not reached at positive values of $x$. Then the minimal values of $p/2k$ correspond at given $m$ to $x=0$ (Fig. 3). This means that the dead zone does not always appear. The development of a dead zone can be seen in Fig. 4. The corresponding data were calculated by (6) with restriction of $0 \leq x \leq 1$. Thus a dead zone always appears at nonzero friction when $2\theta = 90^\circ$, but its appearance takes place only at $m \geq 0.13$ if $2\theta = 105^\circ$ and at $m \geq 0.25$ if $2\theta = 120^\circ$. This result coincides well with our previous finite element simulations of ECAE [8].
Figure 4: Influence of friction and die angle on appearance and relative height of a dead zone

3. DISCUSSION

The obtained results were compared with Segal’s slip lines solution [2, 3]. Corresponding to slip line analysis the relative ECAE pressure can be calculated by formula [2]

\[ \frac{p}{2k} = \left[ \cot \eta + 2(\eta - \theta) \right] + m \left[ \sin \eta \cdot (\sin \eta + \cos \eta) \right] \]

where \( \eta = \pi/2 - 1/2 \cdot \arccos(2m) \).

The summary shear is [3]

\[ \gamma = 2 \cdot \cot \eta + 2 \cdot (\eta - \theta) \]  

The results of comparison are presented in Fig. 5. The good agreement especially for plastic shear was found.

Figure 5: Comparison of upper bound and slip lines solutions at different angels \(2\theta\) and friction factors \(m\)

As expected, the values of relative ECAE pressure obtained by upper bound analysis are slightly higher than corresponding ones derived by slip line theory. The relative divergence of results was evaluated by formula

\[ \delta = \left| \frac{R_{SL} - R_{UB}}{R_{SL}} \right| \cdot 100\% \]

where \(R_{SL}\) and \(R_{UB}\) are values, obtained by slip line and upper bound theory respectively. The evaluation shows that \(\delta\) at two-way \(p/2k\) calculations in the range of \(m \in [0, 0.3]\) is not more than 7.5% at \(\theta=90^\circ\), 9.5% at \(\theta=105^\circ\), and 13.3% at \(\theta=120^\circ\). Thus the maximum disarrangement in relative ECAE pressure does not exceed...
Similar comparison shows that the largest $\delta$ at two-way $\gamma$ calculations in the range of $m \in [0; 0.3]$ is $\delta = 5.5\%$ at $\theta = 90^\circ$, $\delta = 3.75\%$ at $\theta = 105^\circ$ and $\delta = 5.5\%$ at $\theta = 120^\circ$. Thus the maximum disarrangement in summary shear does not exceeds $5.5\%$. Both theories predict sufficient decrease in $p/2k$ and $\gamma$ when $\theta$ angle grows from $90^\circ$ to $120^\circ$. The increase of friction factor leads to rise of the extrusion pressure and to remarkable decrease in total shear.

4. CONCLUSION

The application of the upper bound theory to analysis of ECAE in rectangular and nonrectangular dies allows correctly describe the essential features for this process like appearance of a dead zone and its increase depending on external friction. Physical modeling by plasticine confirms the formation of a dead zone. Also the increase of ECAE pressure and decrease of total plastic shear at rise of friction is well predicted. Finally the decrease of both ECAE pressure and summary shear at growing channel intersection angle is correctly modeled. The upper bound results based on discontinuous velocity field are in good agreement with classical Segal’s slip line solution. The proposed upper bound approach can be applied at further analysis of ECAE in the dies with more complex geometry.

REFERENCES