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Modal Acoustic Impedance of a finite-size spherical source immersed in an unbounded viscoelastic liquid

Mohammad Esmail Aryaee panah[1], Seyyed Mohammad Hasheminejad[2], Farnaz Bagheri[3], Sadeq Malakooti[4]

1 Iran university of science and technology
Narmak, Tehran, Iran
peyman.aryaee@gmail.com
2 Iran university of science and technology
Narmak, Tehran, Iran
hashemi@iust.ac.ir
3 Iran university of science and technology
Narmak, Tehran, Iran
farnaz.bagheriiust@gmail.com
4 Iran university of science and technology
Narmak, Tehran, Iran
s_malakooti@mecheng.iust.ac.ir

Abstract. *A modal impedance matrix is formulated for a spherical source undergoing axisymmetric oscillations in a viscoelastic acoustic medium. The formulation utilizes the governing equations of Verdier and Piau [1] and the analysis method of Hasheminejad and Geers [2]. A matrix based method is used to generate dilatational and translational surface-force curves. At first governing equations of the acoustic fields in the viscoelastic fluid are manipulated to yield the well-known Helmholtz equations and corresponding wave numbers. Equations of radial and meridional velocities and the excess temperature are also derived. In addition, the classical relations for radial and meridional stresses and the heat flux are written as potential based equations. Velocities and the excess temperature as well as the stresses and the heat flux at the surface of the source are expressed as Legendre series with unknown coefficients. The generalized Fourier series solution for the field equations with unknown coefficients are substituted in the potential based equations to find the matrix relations between the coefficients. Frequency dependant modulus and it's relation with the viscosities for the single relaxation time fluid are adopted from Meister et al. [3]. Eventually the modal impedances for the two first modes are calculated and plotted as a function of distance from the source surface. Numerical results for immersion in glycerol as an example are obtained. As it will be explained in the paper, in a limiting case, results of the viscoelastic fluid lead to those of the viscous fluid, hence, as the validation, modal impedance results of the viscoelastic fluid are compared with the results of the viscous fluid of Hasheminejad and Geers [2] and good agreement between the results is observed. Considering the viscoelastic effects (i.e. frequency dependent moduli) can be regarded as the main contribution of the present work to the previous ones (e.g. [2]).*

Keywords: viscoelastic, modal impedance, single relaxation time, spherical source

1 INTRODUCTION

Wave propagation in viscoelastic fluids is of great importance in many applications such as biomechanics. Indeed models that are based on Newtonian fluid properties cannot cover these cases. Consequently many researches in the literature are devoted to this subject. Meister et al in 1960 have investigated the ultrasonic wave properties of associated liquids [3]. Verdier and Piau in 1996 have investigated the acoustic wave propagation in two-phase viscoelastic fluids [1].
Wave propagation investigation is also an efficient way to determine the viscoelastic fluid properties such as shear and compressional moduli. Van est et al in 1997 have used experimental ultrasonic methods to measure viscoelastic properties of fluids [4].
Acoustic impedance is one of the best ways to assess an acoustic source radiation properties. According to the authors knowledge, acoustic radiation impedance of the spherical sources is not investigated in previous works (e.g. [2]), and our goal is to fill this gap.
In this paper equations for velocities and excess temperature and constitutive equations (stress and strain relations) are written in terms of wave potentials. Then the wave potentials are expressed as generalized Fourier series with unknown coefficients. Velocity vector components, the excess temperature, the stresses and heat flux are expanded in Legendre infinite series with unknown coefficients. A frequency dependant viscoelastic relation (relations for complex shear and compressional modulus) is adopted from Meister et al [3]. The complex parameters of the model were used in constitutive and governing equations. At last we obtain the relations for modal acoustic impedance in the viscoelastic fluid.
The modal acoustic impedance is plotted as a function of distance from the source for glycerol as a numerical example.

2 FORMULATION

The set of equations to be written for a continuous medium are the mass, momentum, and energy conservation equations [1]:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0 \tag{1}$$

$$\rho\left(\frac{\partial V}{\partial t} + V \cdot \nabla V\right) = div\,\Sigma \tag{2}$$

$$\rho\left(\frac{\partial u}{\partial t} + V \cdot \nabla u\right) = Tr(\Sigma \cdot D) + \tau \nabla^2 T \tag{3}$$

In which Σ is the stress tensor.
The fluid-particle velocity vector is decomposed to a compression-wave scalar potential $\bar{\phi}$ and a viscous-wave vector potential A as

$$V = -\nabla \bar{\phi} + \nabla \times A \tag{4}$$

Where

$$\bar{\phi} = \phi + \Phi \tag{5}$$

Where ϕ and Φ are and thermal wave scalar potentials respectively.
The governing equations may be manipulated to yield the Helmholtz equations [1]:

$$(\nabla^2 + k^2)\phi = 0$$
$$(\nabla^2 + K_T{}^2)\Phi = 0 \qquad (6)$$
$$(\nabla^2 + \kappa^2)A = 0$$

k corresponds to an adiabatic longitudinal wave, K_T to a thermal wave, and κ to a shear wave. The waves are assumed to be harmonic, so any quantity such as $T(x,t)$ can be written as : $T(x,t) = T(x)e^{-i\omega t}$, the partial differentiation with respect to time gives rise to a multiplication by $-i\omega$: $\dot{T}(x,t) = -i\omega T(x,t)$, so it is convenient to introduce a dynamic shear viscosity $\eta_s{}^*(\omega)$ as a complex function of the frequency, associated with the complex shear modulus $G^*(\omega) = G^{'} + iG^{"}$. Similarly a complex compressional viscosity $\eta_v{}^*(\omega)$ associated with the complex modulus $K^*(\omega) = K^{'} + iK^{"}$ will be used [1]:

$$\eta_s^*(\omega) = \frac{G^*(\omega)}{i\omega}$$
$$\eta_v^*(\omega) = \frac{K^*}{i\omega} \qquad (7)$$

This general theory leads to the Newtonian and elastic behaviors as limiting cases.
In the perfectly elastic solid:

$$\overline{G^*} = \mu \quad \text{and} \quad \overline{K^*} = \lambda + \frac{2}{3}\mu \qquad (8)$$

And in the Newtonian fluid:

$$\overline{G^*} = -i\omega\eta_s \quad \text{and} \quad \overline{K^*} = -i\omega\eta_v \qquad (9)$$

Now we can find the wave numbers as:

$$\frac{1}{k^2} \approx \frac{q^2}{\omega^2}\{1 - iE - (\gamma - 1)iF\}$$
$$\frac{1}{K_T{}^2} \approx -\frac{q^2}{\omega^2} iF \frac{1 - i\gamma E}{1 - iE} \qquad (10)$$
$$\kappa = \omega\sqrt{\frac{\rho_0}{\overline{G^*}(\omega)}}$$

In which

$$E = \frac{(\overline{K^*} + \frac{4}{3}\overline{G^*})i}{\rho_0 q^2}$$
$$F = \frac{\sigma\omega}{q^2} \qquad (11)$$

For a single relaxation time viscoelastic fluid the complex shear modulus is defined as [3]

$$G^*(\omega)=\frac{G_\infty\omega^2\tau_s^2}{1+\omega^2\tau_s^2}+i\frac{G_\infty\omega\tau_s}{1+\omega^2\tau_s^2} \tag{12}$$

And the complex adiabatic modulus of compression is [3]

$$K^*(\omega)=K_0+\frac{K_2\omega^2\tau_v^2}{1+\omega^2\tau_v^2}+i\frac{K_2\omega\tau_v}{1+\omega^2\tau_v^2} \tag{13}$$

For axisymmetric motion in spherical coordinates, $A=(0,0,A)$, and ∇ and ∇^2 are classical. In spherical coordinates the radial and meridional velocities are [2]

$$u_r=-\frac{\partial\bar{\phi}}{\partial r}+\frac{1}{rSin\theta}\frac{\partial}{\partial\theta}(ASin\theta)$$
$$u_\theta=-\frac{\partial\bar{\phi}}{r\partial\theta}-\frac{1}{r}\frac{\partial}{\partial r}(rA) \tag{14}$$

And by using the equation 6 in the governing equations 1, 2 and 3 the excess temperature will be determined as [1]

$$T=g\phi+G\Phi \tag{15}$$

Where

$$g=-\frac{i\omega\gamma}{\beta q^2}+\frac{k^2}{\beta\omega}(i+\gamma E)$$
$$G=-\frac{i\omega\gamma}{\beta q^2}+\frac{{K_T}^2}{\beta\omega}(i+\gamma E) \tag{16}$$

In addition, the classical relations for radial stress, meridional stress, and heat flux may be employed to yield the following potential-based expressions [1]:

$$\sigma_{rr}=e^{-i\omega t}\overline{\eta_s^*(\omega)}((\kappa^2-2k^2)\phi+(\kappa^2-2{K_T}^2)\Phi+2\times\{-\frac{\partial^2\bar{\phi}}{\partial r^2}+\frac{1}{Sin\theta}\frac{\partial}{\partial\theta}[Sin\theta(-\frac{A}{r^2}+\frac{1}{r}\frac{\partial A}{\partial r})]\})$$
$$\sigma_{r\theta}=e^{-i\omega t}\overline{\eta_s^*(\omega)}\{-2\frac{\partial}{\partial\theta}(\frac{\partial\bar{\phi}}{r\partial r}-\frac{\bar{\phi}}{r^2})-(\frac{\partial^2 A}{\partial r^2}-2\frac{A}{r^2})+\frac{1}{r^2}\frac{\partial}{\partial\theta}\{\frac{1}{Sin\theta}\frac{\partial}{\partial\theta}(ASin\theta)\}\} \tag{17}$$
$$q=-\kappa(g\frac{\partial\phi}{\partial r}+G\frac{\partial\Phi}{\partial r})$$

Now the generalized Fourier series solutions for axisymmetric radiating fields in spherical coordinates are [2]

$$\phi(r,\theta)=\sum_{n=0}^{\infty}c_{cn}h_n(kr)P_n(Cos\theta)$$
$$\Phi(r,\theta)=\sum_{n=0}^{\infty}c_{tn}h_n(K_T r)P_n(Cos\theta) \qquad (18)$$
$$A(r,\theta)=-\sum_{n=1}^{\infty}c_{sn}h_n(\kappa r)P_{n,\theta}(Cos\theta)$$

Where $h_n(\xi)$ is the spherical Hankel function of the first kind of order n, $P_n(Cos\theta)$ is the Legendre polynomial of order n, $P_{n,\theta}(Cos\theta)$ is the derivative of $P_n(Cos\theta)$ with respect to θ. From equations 14,15,16,17 then, we may express the velocity, temperature, stress, and heat-flux fields at the surface of the sphere $r=a$ as

$$u_r(a,\theta)=\sum_{n=0}^{\infty}u_{rn}^{a}P_n(Cos\theta)$$
$$u_\theta(a,\theta)=-\sum_{n=1}^{\infty}u_{\theta n}^{a}P_{n,\theta}(Cos\theta)$$
$$T(a,\theta)=\sum_{n=0}^{\infty}T_n^{a}P_n(Cos\theta)$$
$$\sigma_{rr}(a,\theta)=\sum_{n=0}^{\infty}\sigma_{rrn}^{a}P_n(Cos\theta) \qquad (19)$$
$$\sigma_{r\theta}(a,\theta)=-\sum_{n=1}^{\infty}\sigma_{r\theta n}^{a}P_{n,\theta}(Cos\theta)$$
$$q(a,\theta)=\sum_{n=0}^{\infty}q_n^{a}P_n(Cos\theta)$$

The introduction of 18 and 19 into 14, 15 and 17 expressed at r=a, followed by orthogonalization of the resulting equations with respect to the Legendre polynomials, yields the modal surface relations for a single value of n in matrix form as:

$$u_n=R_n(a,h_n)c_n$$
$$P_n=S_n(a,h_n)c_n \qquad (20)$$

Where

$$u_n^T=\{u_{rn}^a\ u_{\theta n}^a\ T_n^a\}$$
$$P_n^T=-\{\sigma_{rrn}^a\ \sigma_{r\theta n}^a\ q_n^a\} \qquad (21)$$
$$c_n^T=\{c_{cn}\ c_{tn}\ c_{sn}\ \}$$

And R_n and S_n are 3×3 matrices to be determined.
The modal acoustic force on a spherical surface is defined by [2]

$$F_n = 2\pi a^2 \int_0^\pi [\sigma_{rr}(a,\theta) P_n(Cos\theta) + \sigma_{r\theta}(a,\theta) P_{n,\theta}(Cos\theta)] Sin\theta d\theta \qquad (22)$$

Introduction of $\sigma_{rr}(a,\theta)$ and $\sigma_{r\theta}(a,\theta)$ and evaluation of the integral then yields

$$F_n = \frac{2\pi a^2}{n+\frac{1}{2}} (\sigma^a_{rrn} + \frac{(n+1)!}{(n-1)!} \sigma^a_{r\theta n}) \qquad (23)$$

3 NUMERICAL RESULTS

Now we are ready to find the modal impedance of a spherical source, in a viscoelastic fluid like glycerol as a numerical example.

A Mathematica code is constructed to calculate the modal acoustic force for a spherical source in glycerol as a function of ka (Fig. 1 and 2). The modal acoustic force is defined by equation 23. Two modes are considered herein, for $n = 0$, F_0 is the dilatational force on the spherical surface , while for $n = 1$, F_1 is the net translational force.

The material-property values which are used in the calculations are listed in table 1 and table 2.

In many applications, the thermal conductivity of the radiator greatly exceeds that of the surroundings acoustic medium; hence, $T(a,\theta)$ was taken as zero.

Fig. 1 and fig. 2 show the inertial and resistive parts of acoustic force of mode 0 and mode 1 for glycerol respectively.

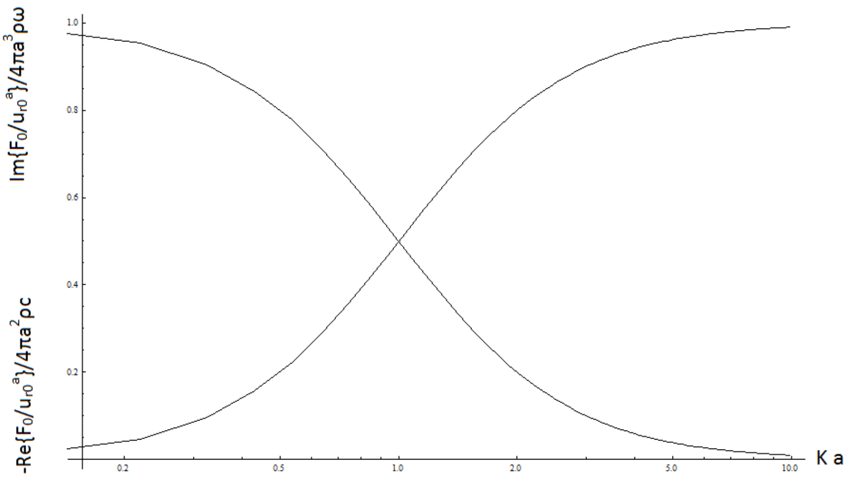

Fig.1. Imaginary and real parts of impedance in glycerol as a viscoelastic fluid for mode 0

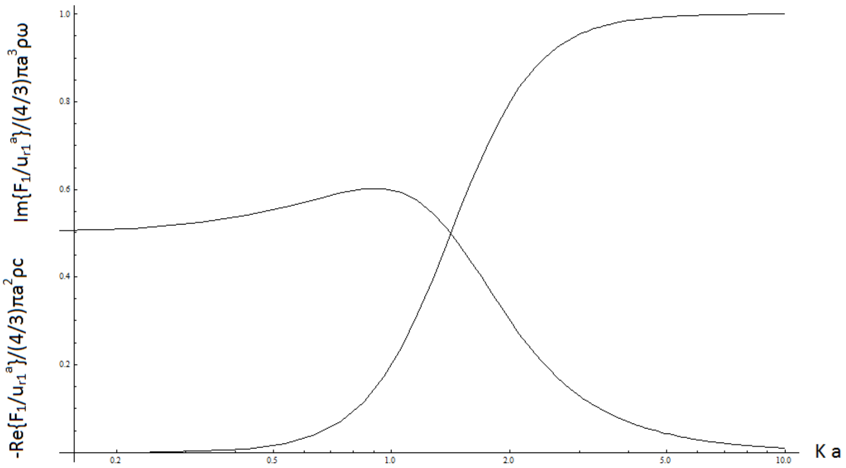

Fig.2. Imaginary and real parts of impedance in glycerol as a viscoelastic fluid for mode 1

Figures 3 and 4 show the modal impedance of modes 0 and 1 for a spherical source in a thermoviscous fluid [2] and a viscoelastic fluid in the limiting case, together, as a validation.

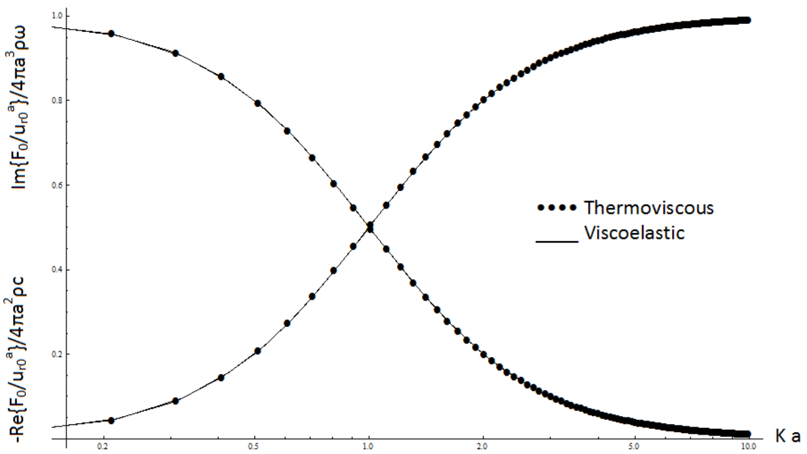

Fig.3. Imaginary and real parts of impedance for mode 0 in glycerol as a viscoelastic fluid (_) in the limiting case and as a themoviscous fluid (•••)

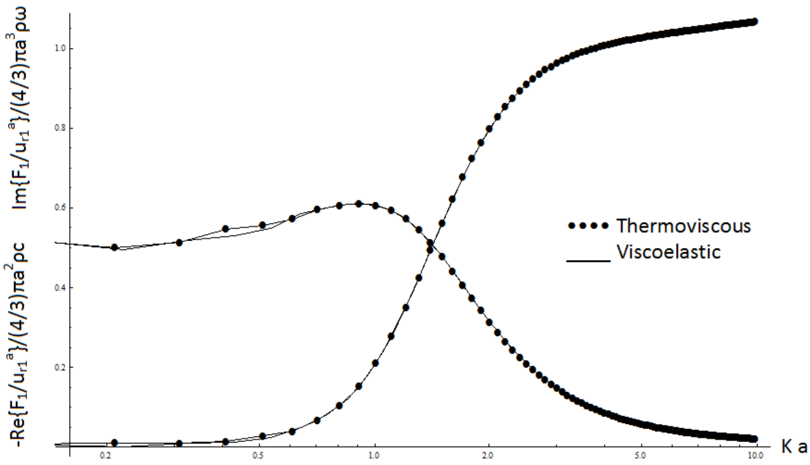

Fig.4. Imaginary and real parts of impedance for mode 1 in glycerol as a viscoelastic fluid (_) in the limiting case and as a themoviscous fluid (•••)

Table 1. Viscoelastic properties of the glycerol [3]

Glycerol	*At* 25 °C
$G_\infty \quad (\frac{N}{m^2})$	0.22925×10^{-10}
$K_\infty \quad (\frac{N}{m^2})$	0.7425×10^{-10}
$K_0 \quad (\frac{N}{m^2})$	0.4605×10^{-10}
$K_2 \quad (\frac{N}{m^2})$	0.282×10^{-10}
τ_s	3548.72×10^{10}
τ_v	3000.3×10^{10}
$\frac{\alpha}{\alpha_{stokes}}$	1.78

Table2 .Thermophysical properties of the glycerol

Glycerol	*At* 25 °C
$q\ (\frac{m}{s})$	1904
$u\ (\frac{W}{mK})$	0.28
$C_P\ (\frac{j}{kgK})$	2430
$\beta\ (\frac{1}{K})$	5×10^{-4}
$\rho_0\ (\frac{kg}{m^3})$	1258.5
γ	1
$\sigma\ (\frac{m^2}{s})$	9.1558×10^{-8}

LIST OF SYMBOLS

u	internal energy of the medium	τ_v	relaxation time for structural rearrangement in
D	symmetrical part of the velocity gradier	G_∞	high frequency shear rigidity
T	temperature	K_2	relaxational part of the modulus
τ	thermal conductivity(assumed to be con	K_0	static compressional modulus
k	complex longitudinal wave number	K_∞	high-frequency compressional modulus
K_T	complex thermal wave number	σ	thermal diffusivity ($\sigma = \frac{\tau}{\rho_0 C_P}$)
κ	complex shear wave number	α	attenuation (nepers/unit dist)
q	adiabatic velocity of sound	η_s	shear viscosity
γ	specific heat ratio	η_v	volume viscosity
ρ_0	density	Cp	specific heat at constant pressure
τ_s	shear relaxation time	β	coefficient of the thermal dilatation

4 CONCLUSION

The most important observations that can be obtained from the figures are:

As it can be seen in figure 1 curves for the resistive and inertial parts of impedance of mode 0 in glycerol coincide in about ka=1 and then begin to diverge. As it is shown in figure 2 the curves of glycerol for the first mode coincide at above ka=1 and then begin to diverge from each other.

Figures 3 and 4 show the modal impedance curves for glycerol as a thermoviscouse fluid [2], and as a viscoelastic fluid in the limiting case together. The results of the limiting case are obtained by using equations 9 (instead equations 12 and 13) in equations 10 and 17.As it can be seen from the figures, very good agreement between the results is observed. As it was shown, in the limiting case, a viscoelastic material acts like a viscous material (or a thermoviscous material when the temperature effects are considered).

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