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Published in:
Advanced Photonics Congress

Publication date:
2012

Document Version
Publisher's PDF, also known as Version of record

Link back to DTU Orbit

Citation (APA):
Sørensen, S. T., Bang, O., Wetzel, B., & Dudley, J. M. (2012). Higher-Order Moment Characterisation of Rogue Wave Statistics in Supercontinuum Generation. In *Advanced Photonics Congress* (pp. JTu5A.22). Optical Society of America.

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Higher-Order Moment Characterisation of Rogue Wave Statistics in Supercontinuum Generation

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Abstract: The noise characteristics of supercontinuum generation are characterized using higher-order statistical moments. Measures of skew and kurtosis, and the coefficient of variation allow quantitative identification of spectral regions dominated by rogue wave like behaviour.
OCIS codes: (030.1640) Coherence; (190.4370) Nonlinear optics, fibers; (320.6629) Supercontinuum generation.

1. Introduction and Theory

There is currently intense research into the noise properties of fiber supercontinuum (SC) generation, motivated both by application demands for low noise broadband sources as well as in the fundamental context of clarifying links with instabilities in other systems. Analysis of SC noise based on the measurement of histograms of energy fluctuations has proven useful to distinguish regimes of Gaussian and long-tailed statistics, allowing spectral regions associated with extreme rogue-like events to be clearly identified [1-2]. Although useful to determine conditions under which such optical rogue waves can be generated or suppressed, the current use of such histogram analysis is exclusively qualitative, making it difficult to use this technique for rigorous comparison between experiment and modelling. Here, we show how histogram analysis of rogue wave like fluctuations can be quantified using the statistical shape descriptors of higher-order central moments (HOM) that quantify not only the mean and variance of a distribution, but also the asymmetry and the presence of long tails. These measures that can be easily derived from experimental photodiode measurements [3] provides a clear and quantitative means of identifying variations at the spectral edges associated with rogue wave events, and yield a complementary tool to phase-dependent coherence measures for interpreting SC noise [4].

Our analysis is based on the statistical framework of central moments used in the study of probability distributions, where we wish to characterize the shape of a particular distribution and not only its location and spread. For a real-valued random variable X, the n-th order central moment around the mean is given by $\mu_n = \langle (X - \langle X \rangle)^n \rangle$ where angle brackets denote an ensemble average. The zeroth and first moments are $\mu_0 = 1$ and $\mu_1 = 0$, respectively, and the second moment is the variance, $\mu_2 = \sigma^2$, where σ is the standard deviation. The normalised third and fourth order moments are of particular interest for revealing non-Gaussian statistics: The *skewness*, $\gamma = \mu_3/\sigma^3$, measures the asymmetry of a distribution and the *kurtosis*, $\kappa = \mu_4/\sigma^4 - 3$, measures the distribution peakedness. Infrequent rogue-like events, associated with qualitative L-shaped statistics are manifested as associated with large skewness and kurtosis. In addition to these moments, a third useful statistical measure is the *Coefficient of Variation*, $C_v = \sigma/\langle X \rangle$, which has the simple interpretation of being inversely proportional to the signal-to-noise ratio.

2. Numerical Simulations and Results

To demonstrate the utility of the HOM descriptors to characterize SC generation, we carried out numerical simulations in the presence of noise to generate a large ensemble of SC spectra. The simulations were based on the generalised nonlinear Schrödinger equation (GNLSE) model which is well-known to accurately reproduce SC spectra and noise properties in agreement with experiments.

The results obtained are given in Fig. 1 and we begin by illustrating the general use of coherence and qualitative histograms of the energy fluctuations of an unstable SC. Fig. 1(a) shows results from an ensemble of 1000 simulations generated from a 20 kW peak power 300 fs (FWHM) sech pulse at 1064 nm propagating through 50 cm of fiber with a zero-dispersion wavelength at 1054 nm. For these parameters the spectral broadening is initiated by noise-seeded modulation instability (MI), which leads to large differences in the resulting SC spectra as seen in the figure.

The unstable nature of the SC in this case is reflected in the fact that there is very low coherence over the full SC bandwidth except in the vicinity of the residual pump. The calculated histograms of pulse energies extracted over a 10 nm bandwidth show a gradual transition from near-Gaussian statistics in the central part of the spectrum to highly skewed non-Gaussian statistics in the spectral wing. But the fact we see zero coherence over most of the SC bandwidth indicates only the presence of severe noise over a wide range, without indicating anything specific about its nature. On the other hand, whilst histograms may be useful for differentiating between Gaussian and non-Gaussian statistics, the selection of particular wavelengths to filter and analyze in this way is not *a priori* clear.

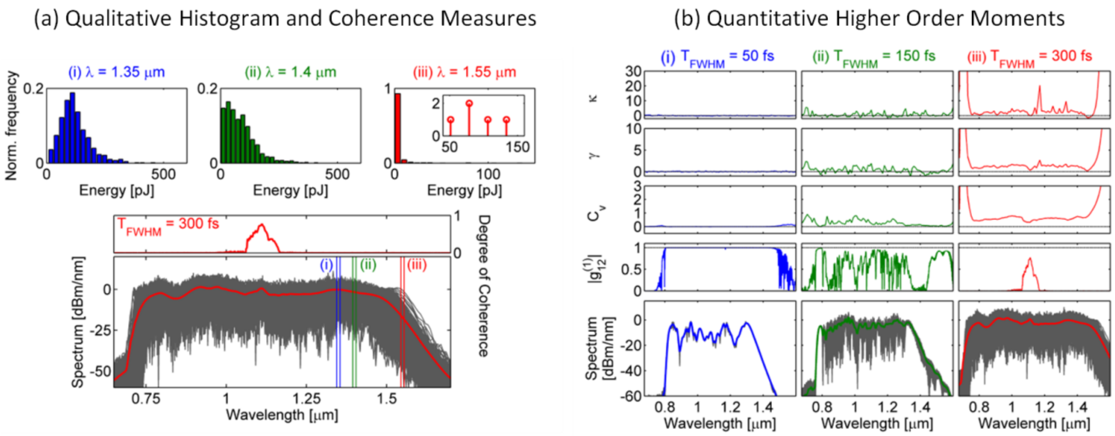

Fig 1: (a) Spectra, and calculated coherence function and histograms for a 300 fs pulse. The energy fluctuation histograms are calculated in the 10 nm windows marked in the spectrum. The inset in (iii) shows a close-up of single high energy pulses. (b) Spectra and coherence ($|g_{12}^{(1)}|$), coefficient of variation (C_V), skewness (γ) and kurtosis (κ) for pulse widths of 50, 150 and 300 fs.

In contrast, we show in Fig. 1(b) how HOMs provide detailed and quantitative characterisation of the probability distribution of spectral fluctuations across the full bandwidth of the SC. Specifically, we show in the figure simulation results with the calculated HOMs (C_v,γ,κ) together with the spectra and coherence for pulse widths of 50, 150 and 300 fs, respectively with all other parameters the same as above. The HOMs are calculated from analysing the full SC spectrum in 10 nm windows. For the 300 fs pulse, the HOMs are generally non-zero across the full SC bandwidth and increase dramatically near both spectral edges. This is a consequence of the noisy solitonic nature of MI-driven SC generation, where intensity fluctuations near the spectral edges arise from large variations in the peak-power of the most red-shifted filtered solitons ejected from the pump. This results in a noisy spectrum with highly non-Gaussian L-shaped statistics near the spectral edges, and we see how this behaviour is very clearly captured by the HOMs.

As the pulse width is decreased, more coherent spectral broadening processes dominate over noise-induced MI, which leads to deterministic pulse break-up and an increasingly coherent spectrum. Indeed, for the shortest 50 fs pulse, we see a completely coherent spectrum with essentially no spectral variations across the full SC, and indeed this is captured by the HOMs that are all zero. A more interesting case is the 150 fs result where we have partial coherence. In this case, it is significant that the spectral coherence is in fact more successful in highlighting spectral structure in the noise characteristics that is not captured by the HOMs. This is an important result which shows the complementary nature of the coherence and HOM approach to SC noise characterization.

3. Conclusion

In conclusion, we have introduced HOMs in the analysis of SC noise and demonstrated how HOMs allow the nature of the SC noise to be accurately quantified across the full bandwidth. The HOMs provide a clear identification of regions of rogue wave behaviour, which we suggest, as a useful guideline, to associate with a product of skew and kurtosis exceeding 10, $\gamma\kappa > 10$. The HOMs and spectral coherence together provide a comprehensive means for analysing SC noise properties and identifying rogue wave statistics, and we therefore suggest adopting this dual approach in future studies of SC noise characteristics.

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