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Quantum Frequency Conversion by Four-wave Mixing Using Bragg Scattering

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Abstract: Two theoretical models for frequency conversion (FC) using nondegenerate four-wave mixing are compared, and their range of validity are discussed. Quantum-state-preserving FC allows for arbitrary reshaping of states for an appropriate pump selection.

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1. Summary

Quantum communication is becoming increasingly important, and an effective method for reshaping and frequency converting (FC) quantum states is essential [1, 2]. Applications include sending a state between nodes (in the 1310 nm or 1550 nm communication window) and detecting the state at a shorter wavelength, or reshaping the temporally wide output from quantum memories to a shorter pulse for better utilization of the existing optical infrastructure [3, 4].

One possible way to FC quantum states while preserving their quantum properties is by four-wave mixing (FWM) in an optical fiber using the nondegenerate process called Bragg scattering (BS), where two strong pumps p and q interact with two sidebands r and s, such that $\pi_p + \pi_s \leftrightarrow \pi_q + \pi_r$, where π_j is a photon at the frequency ω_j, $j \in \{p,q,r,s\}$ [5,6].

We describe FC by BS by the coupled mode equations (CMEs)

$$(\partial_z + \beta_r \partial_t)A_r = i\gamma_{pq}A_s, \qquad (\partial_z + \beta_s \partial_t)A_s = i\gamma_{pq}^* A_r, \tag{1}$$

where β_r and β_s are the group slownesses (inverse group velocities) of the idler and the signal respectively, and $A_r(t,z)$ and $A_s(t,z)$ are the sideband amplitudes whose squares are in units of power. The coefficient γ is proportional to the Kerr nonlinearity coefficient, and for waves that are not too far separated in frequency $\beta_s = \beta_p$ and $\beta_q = \beta_r$ [6]. The coupling function is $\gamma_{pq} = \gamma A_p(t - \beta_s z)A_q^*(t - \beta_r z)$, where the pump shapes are normalized. For simplicity we only consider the Green function (GF) describing the influence on the output idler s from the input signal r, G_{rs}. In the low-efficiency limit, $\gamma/|\beta_r - \beta_s| = \gamma/\beta_{rs} = \bar{\gamma} \ll 1$, one sideband is assumed constant leading to the perturbative solution

$$G_{rs}^{pert.}(t;t') = i\bar{\gamma}A_q^*[t - \beta_r(l - z_i)]A_p[t' + \beta_s z_i]\,H(t' + \beta_r l - t)\,H(t - t' - \beta_s l), \tag{2}$$

where H is the Heaviside step-function and z_i the interaction distance. A remarkable consequence of this result is that for long fibers the step-functions tend to unity and the GF is separable in t and t'. Using standard mathematical methods the CMEs are solvable for an arbitrary strength parameter leading to the exact solution

$$G_{rs}^{exact}(t;t') = i\bar{\gamma}A_q^*(t,l)\,J_0\{2\bar{\gamma}[\xi(t,t')\eta(t,t')]^{1/2}\}A_p(t',0)\,H(t' + \beta_r l - t)\,H(t - t' - \beta_s l), \tag{3}$$

where J_0 is the Bessel function of zeroth order and the effective interaction distances are $\eta(t,t') = \int_{t'}^{t - \beta_s l}|A_p(s)|^2\,ds$ and $\xi(t,t') = \int_{t - \beta_r l}^{t'}|A_q(s)|^2\,ds$. Equations (2) and (3) are valid for arbitrary pump shapes.

The interesting parameter that differentiates the exact GF (3) from the perturbative GF (2), is the strength parameter $\bar{\gamma}$. In the low-efficiency regime the two models agree, whereas for larger values a qualitative different behavior is observed which is confirmed in Fig. 1(a) and (b). The perturbative GF scales linearly with $\bar{\gamma}$, while the exact GF exhibits a complex dependence in $\bar{\gamma}$. The Schmidt coefficients are the square roots of the singular values of the GF [6, OE]. They are useful as they predict separability, and from Fig. 1(c) we see that the perturbative GF is separable in this
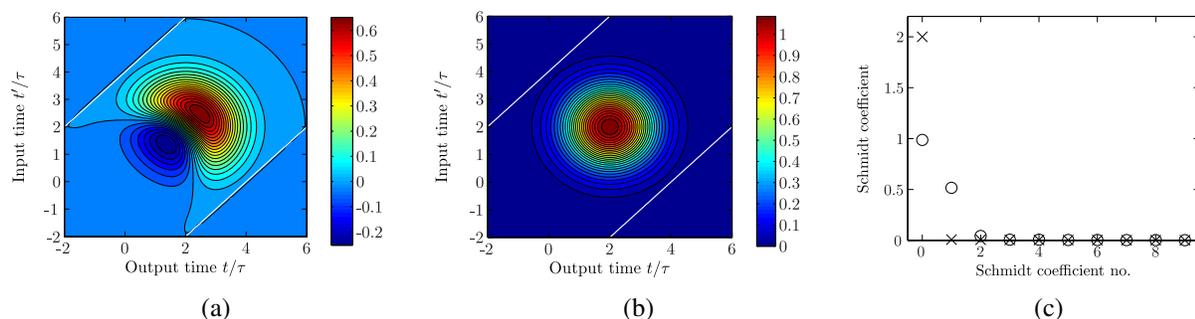
(a)   (b)   (c)

Fig. 1. Plots in the high-conversion regime $\bar{\gamma} = 2$ and $\beta l/\tau = 4$. The white lines mark the area of causality. (a) Contour plot of the exact GF. (b) Plot of the perturbative GF. (c) The first ten Schmidt coefficients, crosses denote the approximate GF.

limit, in contrast to the fact that we have more than one nonzero Schmidt coefficient. All the results are for Gaussian pumps and more interesting pump shapes will be discussed at the meeting.

To quantify the difference between the two GFs, the ratio $\iint |G_{rs}^{exact} - G_{rs}^{pert.}|^2 \, \mathrm{d}t\mathrm{d}t' / \iint |G_{rs}^{exact}|^2 \, \mathrm{d}t\mathrm{d}t'$ was calculated, see Fig. 2(a). For all the considered pump shapes, less than 5 % deviation is possible for $\bar{\gamma} < 0.7$. Consider one particular photonic crystal fiber with $\beta_{rs} \sim 10^{-11}\,\mathrm{s\,m^{-1}}$ this means that the perturbative solution is valid for pump energies of the order $10^{-8}\,\mathrm{J}$ [6]. Figure 2(b) shows the square of the two lowest-order squared Schmidt coefficients for the exact Green function and the squared lowest-order coefficient for the perturbative model. Figure 2(b) does not depend on the pump shape but is general as long as a complete collision has occurred. For $\bar{\gamma} > 1$ complete separability is not possible in the exact model, but this is not captured by the simpler one as it only has one nonzero Schmidt coefficient. However, for 95 % conversion efficiency (λ_0) in the exact model, the first coefficient is nine times larger than the second one, so it is approximately separable. For $\bar{\gamma} = 0.7$ a conversion efficiency of 35 % is achievable.
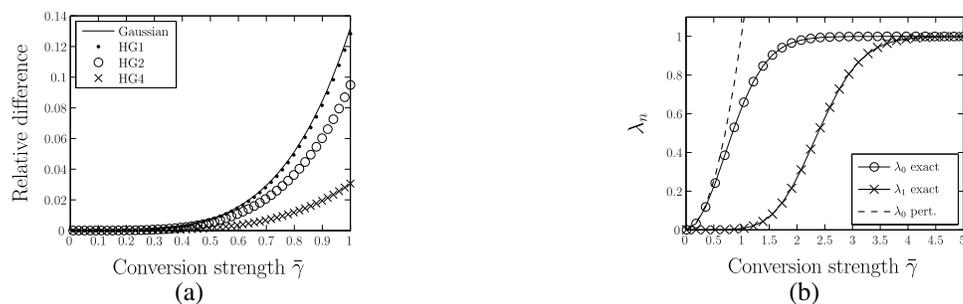

(a)   (b)

Fig. 2. (a) The relative difference between the two models for $\beta l/\tau = 4$ for different pumps where HGn is Hermite-Gauss modes of order n. (b) The square of the Schmidt coefficients for HG0 pumps.

In summary, we showed that the exact solution for frequency-conversion using Bragg scattering describes effects that are not captured by the simpler model, but a good agreement between the two models is possible for $\bar{\gamma} < 0.7$. The problem of self-phase and cross-phase modulation will be considered elsewhere. These effects chirp the Schmidt modes, but their power profiles are similar as those predicted by Eqs. (2) and (3) [6, OE].

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