Benchmarking of optimization methods for topology optimization problems

Susana Rojas Labanda, PhD student
Mathias Stolpe, Senior researcher

11th World Congress on Computational Mechanics. Barcelona 2014
Why?

- Asses general purpose 2nd order optimization methods in topology optimization problems.
Why?

• Asses general purpose 2nd order optimization methods in topology optimization problems.

Main results from the Benchmarking

• GCMMA outperforms MMA.
• GCMMA and MMA tend to obtain a design with large KKT error.
• The performance of GCMMA and MMA do not highlight respect to other solvers.
• The interior-point solver IPOPT, when the exact Hessian is used (IPOPT SAND), produces the best designs using few number of iterations
• IPOPT SAND is the most robust solver in the study.
• The SAND formulation requires lot of memory and computational time.
Topology optimization problems

- **Goal**: Obtain optimal design of a structure with given loads.

Model as an optimization problem

\[
\begin{align*}
\text{minimize} & \quad f(x) \\
\text{subject to} & \quad g(x) \leq 0 \\
& \quad h(x) = 0.
\end{align*}
\]
Topology optimization formulations

**SAND formulation:**

- **Minimum compliance**
  
  \[
  \begin{align*}
  &\text{minimize} & f^T u \\
  &\text{subject to} & a^T t \leq V \\
  & & K(t)u - f = 0 \\
  & & 0 \leq t \leq 1.
  \end{align*}
  \]

- **Minimum volume**
  
  \[
  \begin{align*}
  &\text{minimize} & a^T t \\
  &\text{subject to} & f^T u \leq C \\
  & & K(t)u - f = 0 \\
  & & 0 \leq t \leq 1.
  \end{align*}
  \]

- **Compliant mechanism design**
  
  \[
  \begin{align*}
  &\text{minimize} & l^T u \\
  &\text{subject to} & a^T t \leq V \\
  & & K(t)u - f = 0 \\
  & & 0 \leq t \leq 1.
  \end{align*}
  \]

- \(f \in \mathbb{R}^d\) the force vector.
- \(a \in \mathbb{R}^n\) the volume vector.
- \(V > 0\) is the upper volume fraction.
- \(C > 0\) the upper bound of the compliance.
- \(l \in \mathbb{R}^d\) vector that indicates the output displacement.
Topology optimization formulations

NESTED formulation:

• Minimum compliance
  minimize \( t \) \( u^T(t)K(t)u(t) \)
  subject to \( a^T t \leq V \)
  \( 0 \leq t \leq 1 \).

• Minimum volume
  minimize \( t \) \( a^T t \)
  subject to \( u^T(t)K(t)u(t) \leq C \)
  \( 0 \leq t \leq 1 \).

• Compliant mechanism design
  minimize \( t \) \( l^T u(t) \)
  subject to \( a^T t \leq V \)
  \( 0 \leq t \leq 1 \).

• \( u(t) = K^{-1}(t)f \).
• \( f \in \mathbb{R}^d \) the force vector.
• \( a \in \mathbb{R}^n \) the volume vector.
• \( V > 0 \) is the upper volume fraction.
• \( C > 0 \) the upper bound of the compliance.
• \( l \in \mathbb{R}^d \) vector that indicates the output displacement.
Considerations on the problem formulation

• Use only one external static load.
• Linear elasticity in the equilibrium equation.
• Assume $K(t) \succ 0$ to avoid ill-conditioning.
• Use continuous density variables.
• Use SIMP penalization and a density filter.


Optimization methods

- **OC**: Optimality criteria method.
- **MMA**: Sequential convex approximations.
- **GCMMA**: Global convergence MMA.


Optimization methods

- **OC**: Optimality criteria method.
- **MMA**: Sequential convex approximations.
- **GCMMA**: Global convergence MMA.
- **FMINCON**: Interior-point MATLAB. Use exact Hessian.
- **SNOPT**: Sequential quadratic programming. BFGS approximations.
- **IPOPT**: Interior-point software. Exact Hessian in the SAND formulation, BFGS in the NESTED formulation.


Benchmarking in topology optimization

- **How?** Using performance profiles.
  - Evaluate the cumulative ratio for a performance metric.
  - Represent for each solver, the percentage of instances that achieve a criterion for different ratio values.

\[
\rho_s(\tau) = \frac{1}{n} \text{size}\{p \in P : r_{p,s} \leq \tau\},
\]

\[
r_{p,s} = \frac{\text{iter}_{p,s}}{\min\{\text{iter}_{p,s} : s \in S\}}.
\]

Benchmarking in topology optimization

- **How?** Using performance profiles.
  - Evaluate the cumulative ratio for a performance metric.
  - Represent for each solver, the percentage of instances that achieve a criterion for different ratio values.

\[
\rho_s(\tau) = \frac{1}{n}\text{size}\{p \in P : r_{p,s} \leq \tau\},
\]

\[
r_{p,s} = \frac{\text{iter}_{p,s}}{\min\{\text{iter}_{p,s} : s \in S\}}.
\]

Benchmark set of topology optimization problems

Minimum compliance /minimum volume

- Michell, Cantilever and MBB domains, respectively.

- Length ratio: Michell: $1 \times 1$, $2 \times 1$, and $3 \times 1$. Cantilever: $2 \times 1$, and $4 \times 1$. MBB: $1 \times 2$, $1 \times 4$, $2 \times 1$, and $4 \times 1$.

- Discretization: 20, 40, 60, 80, 100 elements per ratio.

- Volume constraint: $0.1 – 0.5$.

- Compliance constraint: $1$, $1.25$, $1.5 \times C$. Where $C = f^T K^{-1} (t_0) f$.

- Total Problems Compliance: 225.

- Total Problems Volume: 135.
Benchmark set of topology optimization problems

Compliant mechanism design

- Force inverter, Compliant gripper, Amplifier, Compliant lever, and Crimper domain examples, respectively.

- Length ratio: $1 \times 1$ and $2 \times 1$.
- Volume constraint: $0.2 - 0.4$
- Discretization: 20, 40, 60, 80, 100 elements per ratio.
- Total Problems Mechanism Design: 150.
Performance profiles for minimum compliance problems

Objective function value

Number of iterations

Performance profiles in a reduce test set of 121 instances.

Penalization of problems with KKT error higher than $\omega = 1e^{-3}$. 
Performance profiles for minimum compliance problems

Objective function value

Number of iterations

Performance profiles in a reduce test set of 121 instances.

Penalization of problems with KKT error higher than $\omega = 1e - 3$. 
Performance profiles for compliant mechanism design problems

Performance profiles in a reduce test set of 124 instances.

Penalization of problems with KKT error higher than $\omega = 1e^{-3}$.
Performance profiles for compliant mechanism design problems

Objective function value

Number of iterations

Performance profiles in a reduce test set of 124 instances.

Penalization of problems with KKT error higher than $\omega = 1e - 3$. 
Performance profiles for minimum volume problems

Performance profiles in a reduce test set of 64 instances.
Penalization of problems with KKT error higher than $\omega = 1e - 3$. 
Performance profiles for minimum volume problems

Objective function value

Number of iterations

Performance profiles in a reduce test set of 64 instances.
Penalization of problems with KKT error higher than $\omega = 1e - 3$. 
Conclusions

- **Important contributions.**
  - Develop a large topology optimization test set.
  - Introduction to performance profiles in topology optimization.
  - First extensive comparative study of the performance of the state-of-art topology optimization methods with general non-linear optimization solvers.

- What is missing?
  - Large-scale problems, 3D domains, advanced elements,...
  - Other regularization schemes.
  - Different formulations: Displacement constraint, stress constraints,...
  - More optimization solvers.
  - ...

- What can we conclude from the performance profiles?
  - GCMMA outperforms MMA.
  - GCMMA and MMA tend to obtain a design with large KKT error.
  - IPOPT -S produces better designs using few number of iterations.
  - IPOPT -S is the most robust solver in the study.
  - The SAND formulation requires a lot of memory and computational time.
Conclusions

- **Important contributions.**
  - Develop a large topology optimization test set.
  - Introduction to performance profiles in topology optimization.
  - First extensive comparative study of the performance of the state-of-art topology optimization methods with general non-linear optimization solvers.

- **What is missing?**
  - Large-scale problems, 3D domains, advance elements,...
  - Other regularization schemes.
  - Different formulations: Displacement constraint, stress constraints,...
  - More optimization solvers.
  - ...
Conclusions

• **Important contributions.**
  - Develop a large topology optimization test set.
  - Introduction to performance profiles in topology optimization.
  - First extensive comparative study of the performance of the state-of-art topology optimization methods with general non-linear optimization solvers.

• **What is missing?**
  - Large-scale problems, 3D domains, advance elements,...
  - Other regularization schemes.
  - Different formulations: Displacement constraint, stress constraints,...
  - More optimization solvers.
  - ...

• **What can we conclude from the performance profiles?**
  - GCMMA outperforms MMA.
  - GCMMA and MMA tend to obtain a design with large KKT error.
  - IPOPT-S produces better designs using few number of iterations
  - IPOPT-S is the most robust solver in the study.
  - The SAND formulation requires lot of memory and computational time.
THANK YOU !!!

This research is funded by the Villum Foundation through the research project Topology Optimization – the Next Generation (NextTop).