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1 Introduction

High School Student Sectioning (HSSS) is a planning problem each Danish high school faces every year. There has been an increase of scientific research within Education Timetabling and High School Timetabling the last decade due to the three International Timetabling Competitions. However, many of the articles are concerned with the actual timetabling problem and not the sectioning of students. The HSSS is closely related to the Elective Course Student Sectioning (ECSS) (Kristiansen et al (2011)). Both problems are yearly recurrent planning problems at the Danish high schools. But where ECSS is concentrated on assigning 2nd and 3rd year students to elective courses given their requests, the HSSS is concerned with the partitioning of the 1st year students to form cohort. The overall goal of the HSSS is to create a solution method which can be implemented in the cloud based high school administration software Lectio and hence available to the majority of Danish high schools. The purpose of this abstract is to create an IP model for the problem and try to solve it using the state-of-art MIP solver Gurobi.

2 High School Student Sectioning

The first problem we consider is the partitioning of students to form cohorts, where a cohort of students will be timetabled as one entity for most of their classes. Students are distinguished by their choice of specialization, where some possible specializations for linguistic students are listed in Table 1. Each student

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chooses exactly one specialization. In this example, the school has specified that we need to form 5 cohorts each containing no more than 30 students where each cohort contains students from one or more specializations. We wish to minimize the number of specializations in each cohort. Table 1 shows a possible solution.

Table 1 An example of forming cohorts for linguistic students who have chosen one of 4 linguistic specializations. The upper limit on cohort size is 30 students

Specialization	# Students	Cohort 1	Cohort 2	Cohort 3	Cohort 4	Cohort 5
English-German	18		18			
English-Spanish	40	28	12			
English-Music	9			9		
English-SocialSci	77			20	30	27

The example above considered students each of whom has chosen a specialization in the arts area. A similar independent problem needs to be solved for students selecting a specialization within science.

As well as choosing a specialization, each student chooses one Linguistic subject, such as French or Italian, and one Arts subject such as Drama or Media. After the partitioning of students to cohorts, we wish to assign the cohorts to time-slots. One for each type of subject. There will typically be many cohorts assigned to the same time-slot. These assignments then determine the number of students taking each subject, such as French or Italian, in each time-slot, and elective course classes can then be created based on these numbers.

The primary objective of the HSSS is to assign all the students to a cohort, while minimizing the number of specializations represented in each cohort. As second objective, we want to minimize the amount of classes needed to fulfill all course requests and to minimize the number of courses represented in each time-slot.

2.1 IP model

For the partitioning of students to cohorts at a high school we have a set of students, a set of specializations $\mathcal{S}$, a set of cohorts $\mathcal{C}$. To avoid some of the symmetry which the problem contains, we introduce the set of student groups, $\mathcal{G}$. A student group consists of students which have requested exactly the same specialization and elective courses. $L_c \in \mathbb{N}$ and $U_c \in \mathbb{N}$ are the lower and upper limit on the cohort size, respectively. Let $F_g \in \mathcal{N}$ be the size of the student group g, and hence the flow demand of student group g. And let $A_{g,g'}$ be 1 if the student groups g and g' are not allowed to be in the same cohort. The decision variable $x_{g,c} \in \{0,1\}$ takes value 1 if there exists a flow from student group g to cohort c, zero otherwise. The variable $\bar{x}_{g,c} \in \mathbb{N}$ is the amount of flow from student group g to cohort c.

$$\min \alpha \cdot \sum_{g,c} x_{g,c} + \beta \cdot \sum_{g} \bar{x}_{g,c_\emptyset} \tag{1}$$

$$\text{s.t.} \quad \sum_{c} \bar{x}_{g,c} + \bar{x}_{g,c_\emptyset} = F_g \qquad \forall\, g \tag{2}$$

$$\bar{x}_{g,c} \leq \min[F_g, U_c] \cdot x_{g,c} \,\, \forall\, g, c \tag{3}$$

$$\sum_{g} \bar{x}_{g,c} \quad\quad \le U_c \quad\quad\quad \forall c \quad\quad\quad (4)$$

$$\sum_{g} \bar{x}_{g,c} \quad\quad \ge L_c \quad\quad\quad \forall c \quad\quad\quad (5)$$

$$x_{g,c} + x_{g',c} \quad \le 1 \quad\quad\quad \forall c, g, g', A_{g,g'} = 1 \quad (6)$$

$$x_{g,c} \in \{0,1\} \quad\quad\quad\quad\quad\quad\quad\quad\quad (7)$$

$$\bar{x}_{g,c} \in \mathcal{N} \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (8)$$

Constraints (2) ensure that all students in a student group are assigned a cohort. Constraints (3) ensure that the maximum flow from student group g into a given cohort c, cannot be greater that the upper cohort size or the size of the student group. Constraints (4) and (5) make sure that the total flow into a cohort is limited by the upper and lower cohort size. Finally constraints (6) make sure that two student groups which are not allowed to be placed in the same cohort are not done so.

The second part of the HSSS is the elective course assignment where we want to assign the cohort to one time-slot, $\mathcal{T}$ of each course type, $\mathcal{D}$. The types are the linguistic and the artistic courses. Let $T_d \subseteq \mathcal{T}$ be the set of times used for course of type d. $R_{g,e}$ denotes whether student group g has requested elective course e.

The decision variable $y_{c,t,d} \in \{0,1\}$ denote whether cohort c are assigned time-slot t of course type d. And let variable $\bar{v}_{e,t} \in \mathcal{N}$ be the number of course classes needed for elective course e in time slot t. The parameter $K_{e,t} \in \mathcal{N}$ denote the maximum number of classes of course e which is possible to create in time t.

The objective of this part is to minimize the number of course classes and the following constraints are imposed.

$$\min \gamma \cdot \sum_{e,t} \bar{v}_{e,t} \quad\quad\quad\quad\quad\quad\quad\quad\quad (9)$$

$$\text{s.t.} \sum_{t \in T_d} y_{c,t,d} \quad\quad\quad = 1 \quad\quad \forall c, d \quad\quad (10)$$

$$\sum_{c} y_{c,t,d} \quad\quad\quad \ge 1 \quad\quad \forall t, d \quad\quad (11)$$

$$\sum_{g} R_{g,e} \cdot \bar{x}_{g,c} - M(1 - y_{c,t,d}) \le U_e \cdot \bar{v}_{e,t} \quad \forall c, e, t, d \quad (12)$$

$$\bar{v}_{e,t} \quad\quad\quad \le K_{e,t} \quad\quad \forall e, t, K_{e,t} > 0 \quad (13)$$

$$y_{c,t,d} \in \{0,1\} \quad\quad\quad\quad\quad\quad\quad\quad\quad (14)$$

$$\bar{v}_{e,t} \in \mathcal{N} \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (15)$$

Constraints (10) make sure that every cohort is assigned exactly one time slot for each course type. Constraints (11) ensure that there is at least one cohort in each time slot, whereas Constraints (12) sets the value for variable $\bar{v}_{e,t}$. Constraints (13) ensure that the recourse limitation for a given course at a given time is respected.

By making small adjustment on how the cohorts are formed and in which time slots they are placed, it is possible to improve the distribution and then ease the problem of creating classes and the entire timetable problem. This is done by minimizing the number of specializations and courses represented in each cohort, and by minimizing the number of courses represented in each time slot. Let the parameters $D_{g,s}$ and $R_{g,e}$ denote whether the student group g has requested

specialization s and course e, respectively. The variables $z_{s,c} \in \{0,1\}$ and $z_{e,c} \in \{0,1\}$ denote whether specialization s or course e is represented in cohort c, respectively. And let $v_{e,t} \in \{0,1\}$ take value 1 if course e is presented in time slot t.

$$\min \sum_{s,c} \delta_s \cdot z_{s,c} + \sum_{e,c} \delta_e \cdot z_{e,c} + \sum_{e,t} \delta \cdot v_{e,t} \tag{16}$$

$$\text{s.t.} \quad D_{g,s} \cdot x_{g,c} \qquad \leq z_{s,c} \ \forall\, g,c,s \tag{17}$$

$$R_{g,e} \cdot x_{g,c} \qquad \leq z_{e,c} \ \forall\, g,e,c \tag{18}$$

$$z_{e,c} + y_{c,t,d} - 1 \leq v_{e,t} \ \forall\, e,c,d \tag{19}$$

$$z_{s,c} \in \{0,1\} \tag{20}$$

$$z_{e,c} \in \{0,1\} \tag{21}$$

$$v_{e,t} \in \{0,1\} \tag{22}$$

Constraints (17) and (18) sets the value for the variables $z_{s,c}$ and $z_{e,c}$ and hence denote if specialization s or course e is represented in cohort c, respectively. Constraints (19) denote whether elective course e is represented in time slot t, using variables $v_{e,t}$.

3 Solution Method and outlook

Currently the model has been tested using Gurobi and 15 real life datasets provided by MaCom A/S. MaCom A/S is the owner of Lectio and they have access to datasets from more than 200 Danish high schools. The results shows that for large instances Gurobi has a hard time finding good solutions within the given running time of 60 seconds. I.e. more work is needed before an implementation of HSSS in Lectio.

References

Kristiansen S, Srensen M, Stidsen TR (2011) Elective course planning. European Journal of Operational Research 215(3):713 – 720, DOI 10.1016/j.ejor.2011.06.039