



## Available fluid codes for turbulence study

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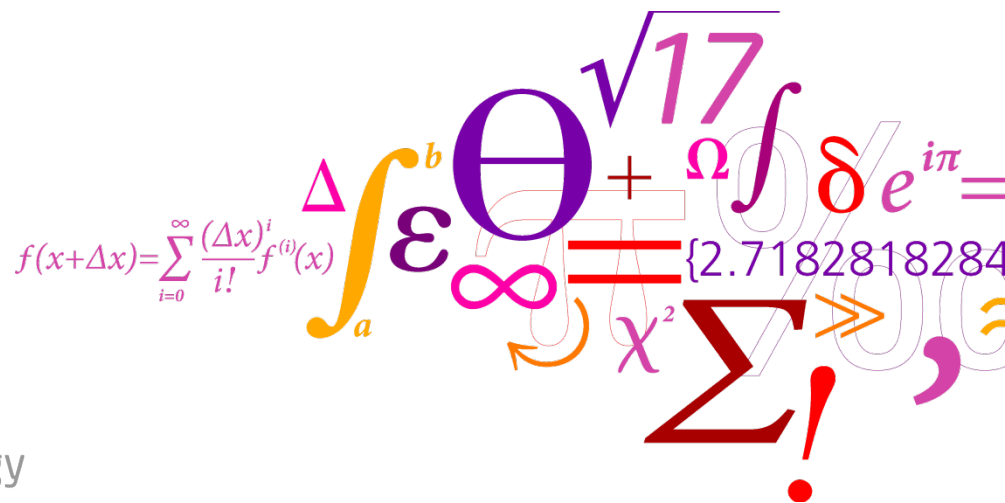
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# Available fluid codes for turbulence study

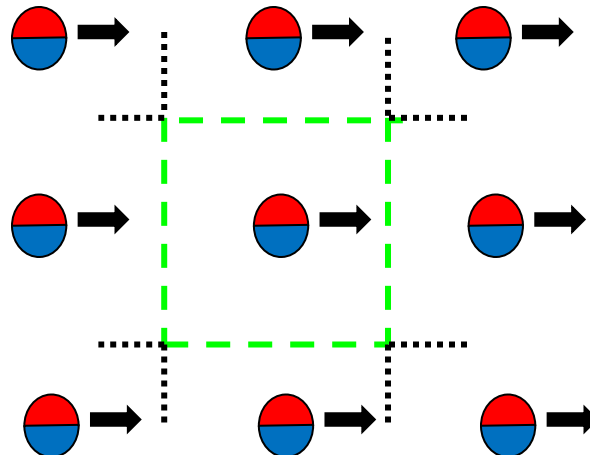
Anders H. Nielsen

Jens Juul Rasmussen, Volker Naulin, Odd Erik Garcia



# Periodic codes (spectral)

- Probably the simplest codes to make
- Easy and fast to develop, a master study
- Easy, fast and compact to run, a bachelor study
- Can use fairly high number of modes on a single CPU: 2048x2048
- Can reach fairly high Reynolds numbers:  $Re(2D) = UL/\mu < 20.000$
- Can fairly easy be parallelized using MPI: linear speedup using 100 CPUs on a 2048x2048 grid
- Note that the domain is infinite with a periodic restriction!



# Periodic codes (spectral)

- Solutions are expanded into Fourier modes (global)

$$\begin{pmatrix} \omega(x, y, t) \\ \psi(x, y, t) \end{pmatrix} = \sum_m \sum_n \begin{pmatrix} \omega_{mn}(t) \\ \psi_{mn}(t) \end{pmatrix} \exp\left(\frac{2\pi imx}{L_x}\right) \exp\left(\frac{2\pi iny}{L_y}\right)$$

- Vorticity equation

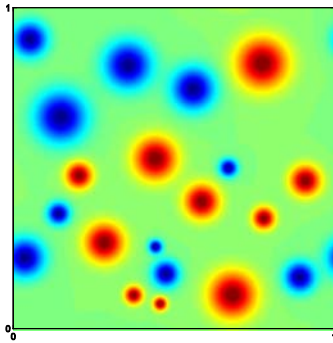
$$\frac{\partial \omega}{\partial t} + J(\psi, \omega) = \nu \nabla^2 \omega \Rightarrow \forall(mn) : \frac{\partial \omega_{mn}}{\partial t} + [\psi, \omega]_{mn} = \nu \nabla^2 \omega_{mn}$$

$$[\psi, \omega]_{mn} = FT \left\{ \frac{\partial \psi}{\partial x} \frac{\partial \omega}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial \omega}{\partial x} \right\}_{mn}$$

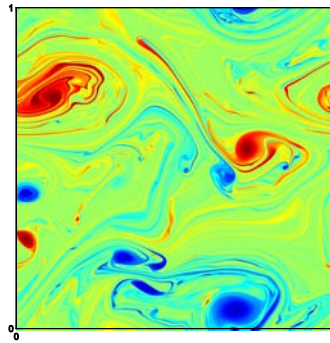
- Fast Fourier Transformation will take 75 % of computational time
- De-aliasing scheme, zero pad the largest 1/3 of the modes
- The Poisson equation is trivial:  $k^2 \psi_k = \omega_k$

# The Risø Euler code

Time = 0.0

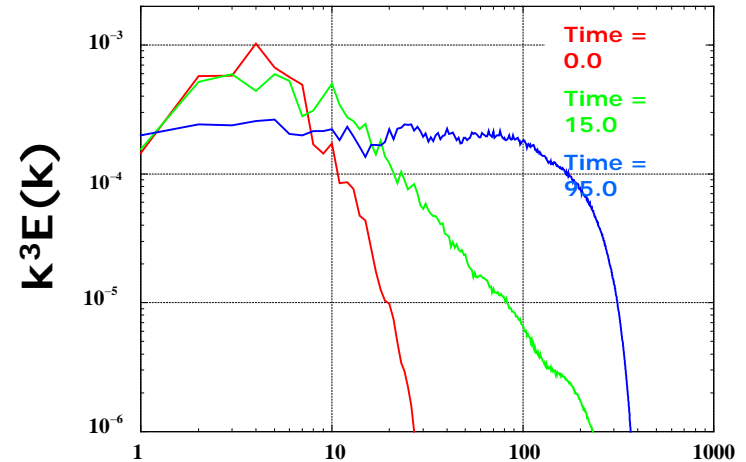


Time = 95.0



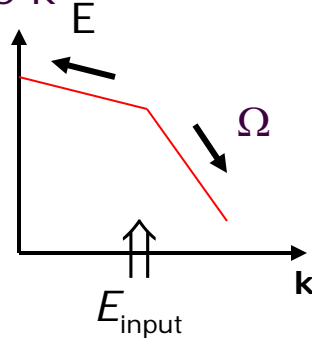
Kuznetsov et al POP **19**, 105110 2007

$$E(k) = \sum_{m,n} u_{mn}^2(t) + v_{mn}^2(t)$$



Inverse cascade :

$$\boxed{\Omega_k = k^2 E_k} \Rightarrow \begin{cases} E \rightarrow \text{small } k \\ \Omega \rightarrow \text{large } k \end{cases}$$



- 1024x1024 points
- 512x512 modes
- de-aliased removes upper 1/3
- $K_{\max} = 340$
- Taken account for viscosity leave us with approximately 2 decades!



Diffusion equation

$$\frac{\partial \omega}{\partial t} = \nabla \cdot (D(\vec{x}, t) \nabla \omega) + \dots \Rightarrow$$

$$(1 - \Delta t \nabla \cdot (D(\vec{x}, t) \nabla)) \omega(t + \Delta t) \approx \omega(t) + \dots$$

Generally gives a complicated matrix, has to be solved by iteration (Petsc)

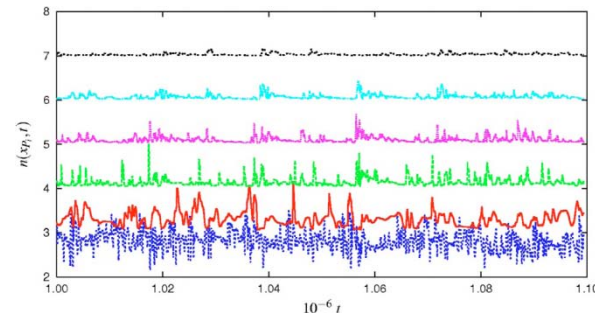
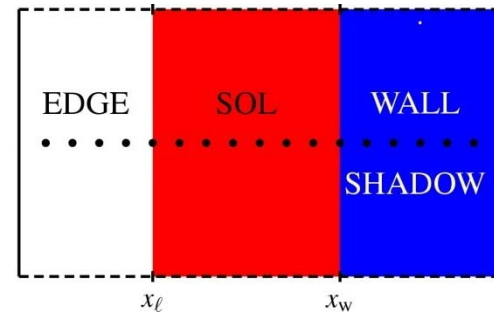
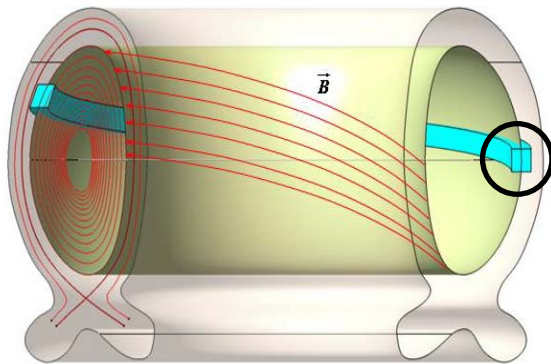
$D = D_0$ : Helmholtz equation

$$(1 - \Delta t D_0 \nabla^2) \omega(t + \Delta t) = \omega(t)$$

banded matrix, Gauss elimination possible

# Finite difference, ESEL

- Global model with self-consistent profiles
- Simulation domain include both edge, SOL and limiter shadow regions
- 2D approximation; parallel loss mechanism modeled by a parameterize loss term
- Input; basic plasma parameters
- Test on TCV, JET, ASDEX with reasonable results
- Collisional diffusion coefficients and parallel loss terms from first principal
- It is a very simple 2D model!



Several millions time point



# Finite difference, ESEL

Interchange model

$$\frac{dn}{dt} + n \square (\phi) - \square (Tn) = \Lambda_n$$

$$\frac{dT}{dt} + \frac{2}{3} T \square (\phi) - \frac{7}{3} T \square (T) + \frac{2}{3} \frac{T^2}{n} \square (n) = \Lambda_T$$

$$\frac{d\omega}{dt} + \square (nT) = \Lambda_\omega$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{1}{B} \mathbf{b} \times \nabla \phi \cdot \nabla$$

$$\frac{1}{B} = 1 + \frac{r_o + \rho_s X}{R_0}$$

$$\square (B, f) = \frac{\partial B}{\partial x} \frac{\partial f}{\partial y} - \frac{\partial B}{\partial y} \frac{\partial f}{\partial x} \quad \left( = \frac{\rho_s}{R_0} \frac{\partial f}{\partial y} \right)$$

$$\Lambda_\alpha = -\frac{\alpha}{\tau_{\square, \alpha}} + D_{\perp, \alpha} \nabla_{\perp}^2 \alpha$$

Subsonic advection:

$$\tau_{\square, n} \square \tau_{\square, \omega} \square \frac{M_{\square} c_s}{L_p}, \quad M_{\square} = 0.5$$

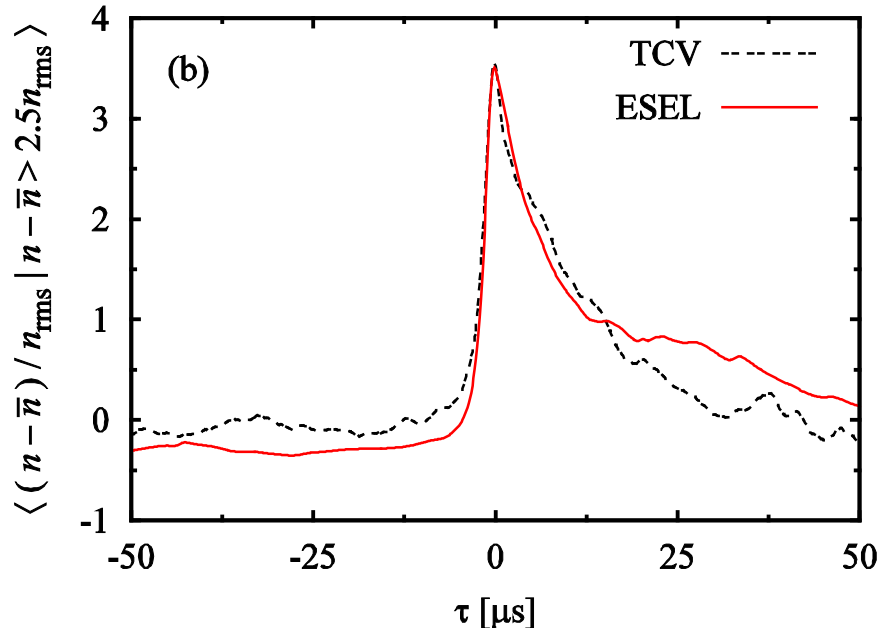
Spitzer-Harm diffusion:

$$\tau_{\square, T} \square \frac{3L_p^2}{2\chi_{\square e}}$$

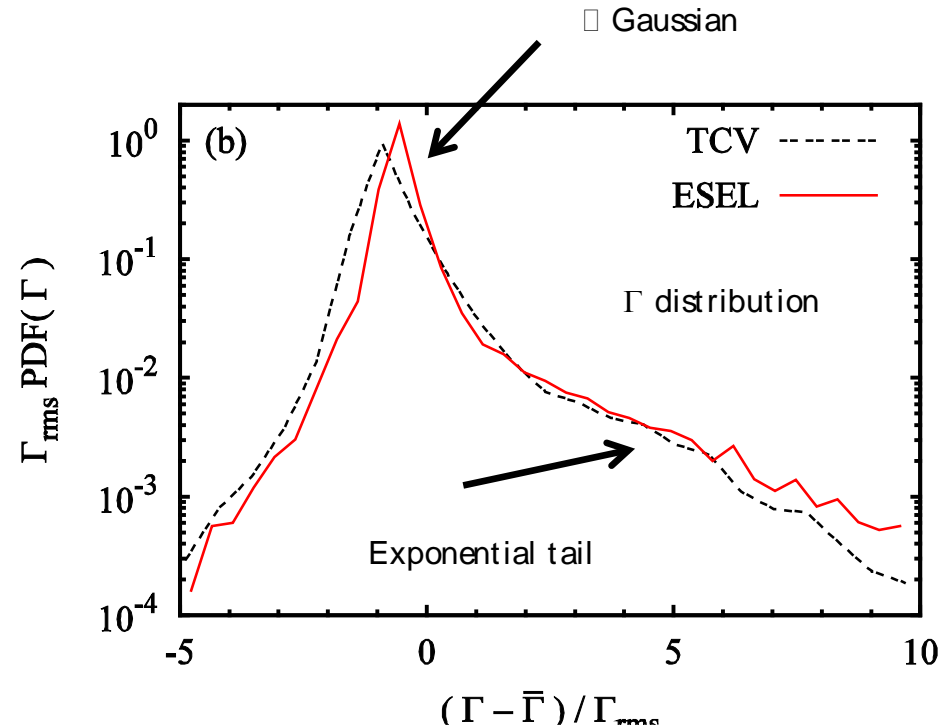
Collisional SOL limit

$$10 < \nu_e^* = \frac{L_{\square}}{\lambda_e} < 80$$

# TCV-ESEL Comparison



Conditionally averaged density blob structure



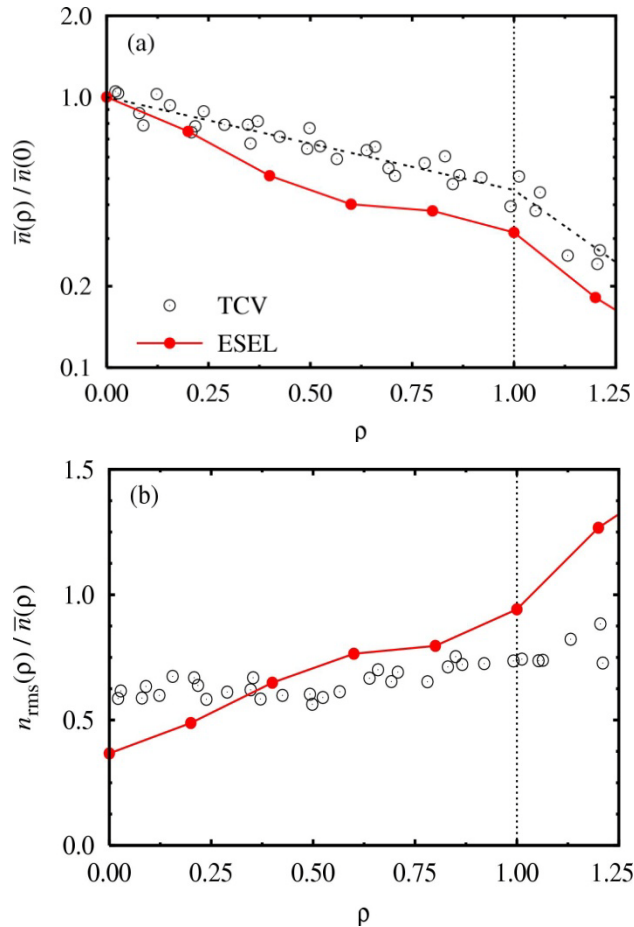
PDF of particle density flux

**Direct comparison with experimental results from the TCV-Tokamak, Lausanne: excellent quantitative agreement**

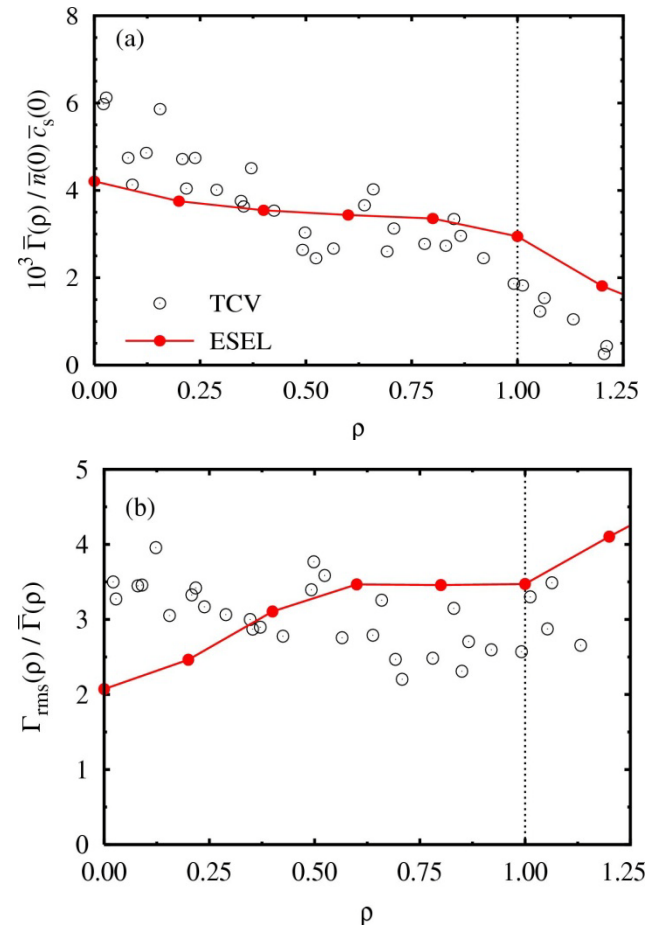
Garcia *et al.* PPCF **48**, L1 (2006)

# TCV-ESEL Comparison

Density profile and relative fluctuations



Particle flux profiles



Good agreement between experiment and turbulence simulations

Garcia *et al.* PPCF **48**, L1 (2006)

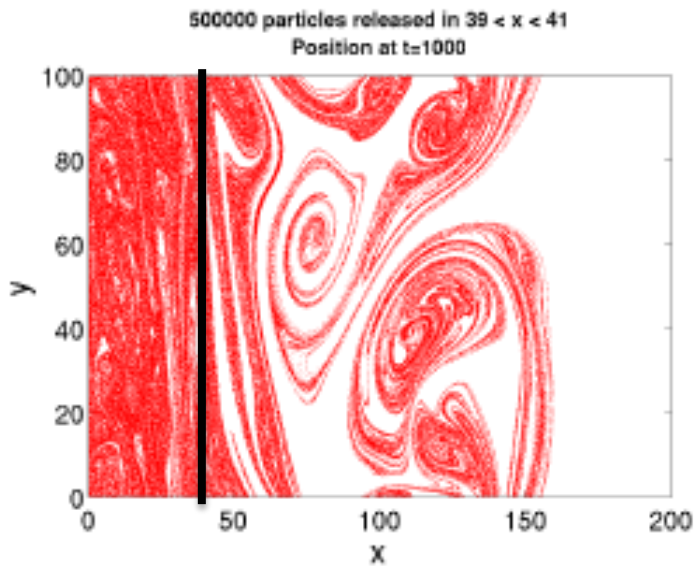


Fig 4: Particles released inside LCFS,  $t = 1000$

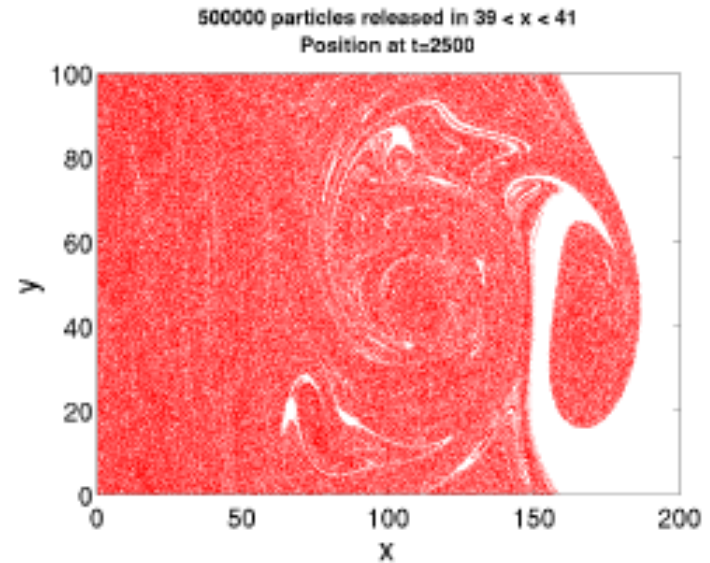
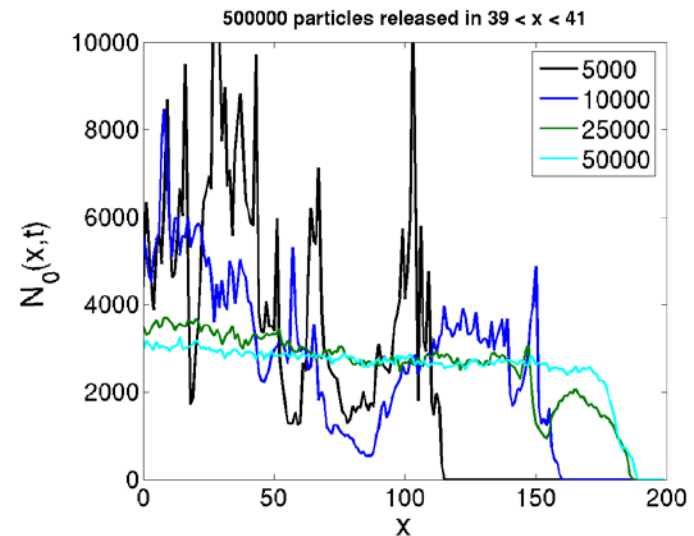


Fig 5: Particles released inside LCFS,  $t = 2500$

Passive particles

$$x(t) = x(0) + \int_0^t v(x, t) dt$$

Summer student S. Boudaux, (2005)



# Solid boundaries - Disk

- Radial points are cosine distributed
- Chebyshev polynomials calculated via cosine transformation
- The Poisson equation decouples in  $\theta$ , a series of banded 1D problem to be solved in  $r$
- $r=0$  should be a regular point
- As  $r \rightarrow 0$  the grid spacing in  $\theta$  decreases:  $1/(2\pi r)$
- Even though these scales are well below the viscosity scale they are extremely unstable and have to be removed manually (zero pad).

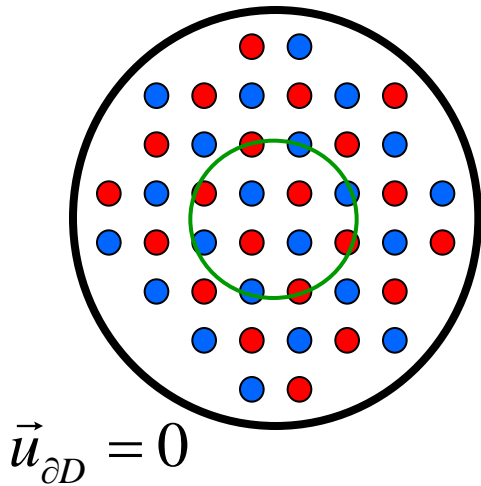
$$\begin{pmatrix} \omega(r, \theta, t) \\ \bar{u}(r, \theta, t) \\ \psi(r, \theta, t) \end{pmatrix} = \sum_m \sum_n \begin{pmatrix} \omega_{mn}(t) \\ \bar{u}_{mn}(t) \\ \psi_{mn}(t) \end{pmatrix} T_m(r) \exp(-in\theta)$$

$$r \in [-1: 1], \theta \in [0: 2\pi]$$

$$T_n(x) = \cos(n \cos^{-1}(x)) = \cos(nz)$$

$$T_0(x) = 1, \quad T_1(x) = x, \quad T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$$

$$T_n(-1) = (-1)^n, \quad T_n(1) = 1$$



Poisson equation

$$\nabla^2 \phi = \omega \Rightarrow$$

$$\forall k: \frac{\partial^2 \phi_k}{\partial r^2} + r \frac{\partial \phi_k}{\partial r} - k^2 \phi_k = r^2 \omega_k \Rightarrow$$

$$\forall k: \underline{\underline{A_k}} \phi_k = \underline{\underline{\omega_k}} + \underline{\underline{BC}}$$

$$\underline{\underline{A_k}} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ x & x & x & x & x & & & & & \\ & x & x & x & x & x & & & & \\ & & x & x & x & x & x & & & \\ & & & & & & \ddots & & & \\ & & & & & & & & & \ddots \\ & & & & & & & & x & x & x & x & x \\ & & & & & & & & & x & x & x & x \\ & & & & & & & & & & x & x & x \end{pmatrix}$$

Energy Spectrym:  $E(r, n) = \frac{r}{2} \sum_n u_n^2(r, t) + v_n^2(r, t)$

# Spectral versus finite difference

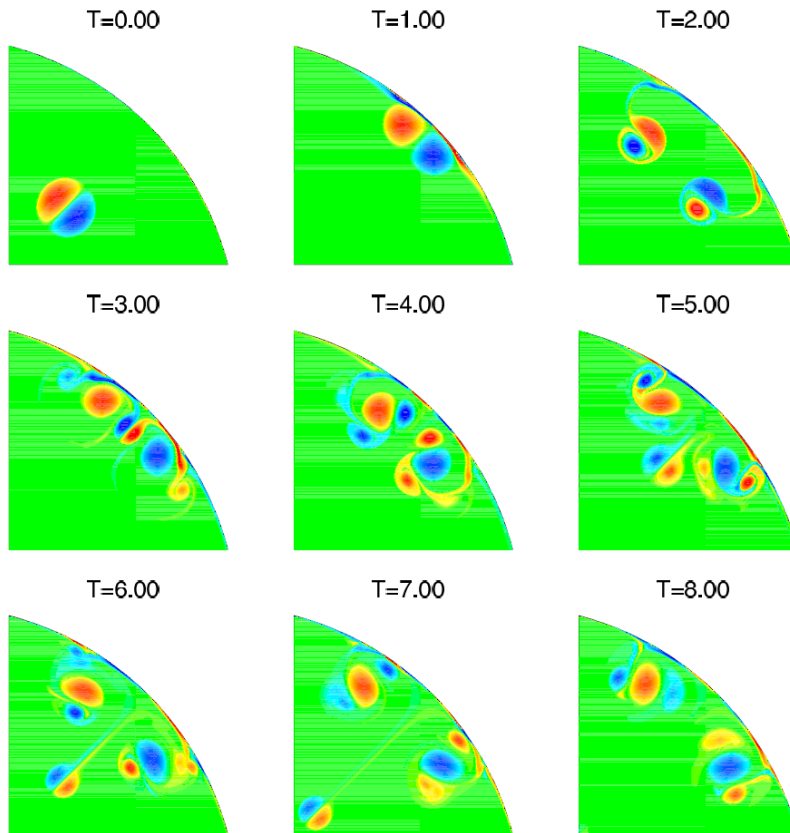


FIG. 5.4. Time evolution of the vorticity field for the interaction of a Lamb-dipole with a no-slip wall. The spectral scheme has been used with  $M = N = 1024$  and  $Re = 2,000$ . Notice that only a part of the computational domain is displayed.

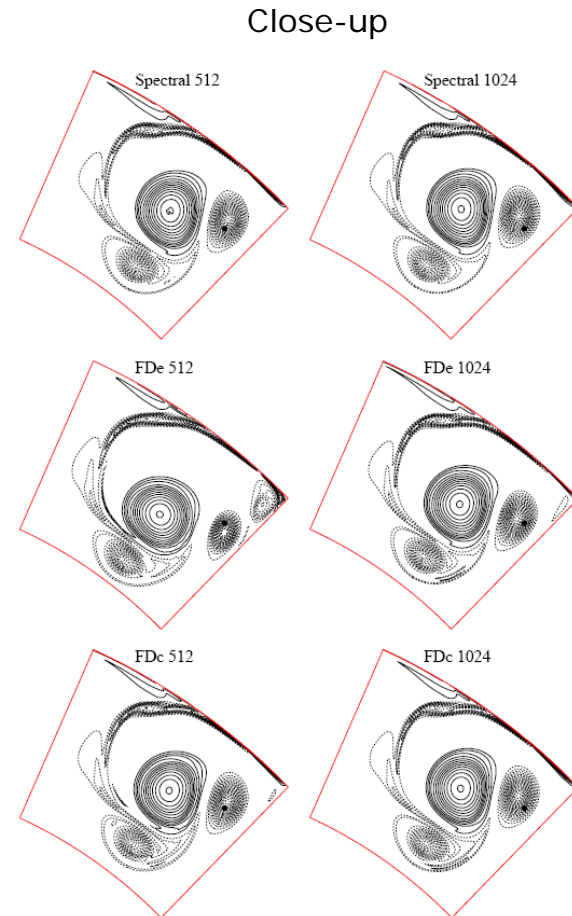


FIG. 5.6. A close-up of vorticity contours for runs with the same parameters as in Figure 5.4 at  $T = 4.0$  and  $Re = 2,000$ . Top: Spectral scheme. Middle: Arakawa scheme using an equidistant radial grid. Bottom: Arakawa scheme using cosine distributed radial grid points. Left resolution 512 and right resolution 1024. The dot in each frame locates the position where the time development is compared; see Figure 5.7.

(V. Naulin and A.H. Nielsen **25**, 104–126 SIAM J. SCI. COMPUT 2003 )

# Spectral versus finite difference

Solve the vorticity equation with solid boundaries in annulus geometry

$$\frac{\partial \omega}{\partial t} + [\omega, \psi] = \nu \nabla^2 \omega, \quad \nabla^2 \psi = -\omega, \quad \bar{u}|_{\partial D} = 0$$

We used a spectral code (Chebyshev-Fourier expansion) and finite difference code (cosine distributed radial points). A Lamb dipole

$$\omega = \begin{cases} \frac{2\lambda U}{J_0(\lambda R)} J_1(\lambda r) \cos(\theta) & , r \leq R \\ 0 & , r > R \end{cases}$$

was used as initial condition and let it interact with the outer wall for different Reynolds numbers,  $Re = UL/\nu$ .

## Conclusion:

- Spectral schemes are more accurate than FD using the same resolution BUT
- Using the same computer power we can obtain similar results for the two different schemes

Spectral

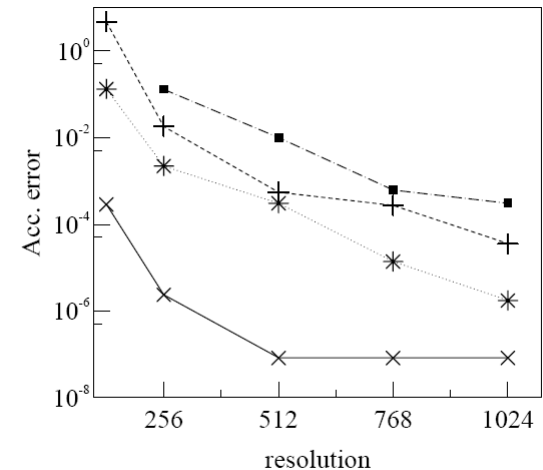


FIG. 5.9. Integrated error calculated from (5.2) versus resolution for the spectral scheme. Reynolds number: 200 (solid line), 1,000 (dotted line), 2,000 (dashed line), 4,000 (dashed-dotted line).

Finite difference

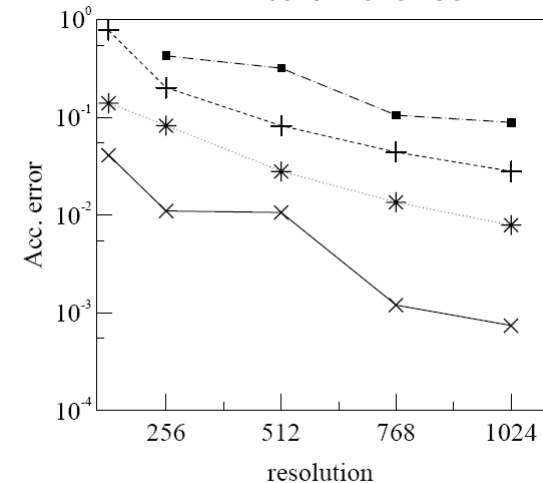


FIG. 5.10. Integrated error calculated from (5.2) versus resolution for the FDe scheme. Reynolds number: 200 (solid line), 1,000 (dotted line), 2,000 (dashed line), 4,000 (dashed-dotted line).

(V. Naulin and A.H. Nielsen **25**, 104–126 SIAM J. SCI. COMPUT 2003 )

# DIESEL

- Global version of "ESEL"
- Covers the full toroidal domain

$$\frac{\partial n^i}{\partial t} + \{\phi, n^i\} = \mu_n \nabla_{\perp}^2 n^i + c_s \nabla_{\parallel} n$$

$$\frac{\partial \omega^i}{\partial t} + \{\phi^i, \omega^i\} = \left\{ \frac{1}{B}, n^i \right\} + \mu_{\omega} \nabla_{\perp}^2 \omega^i + V_A \nabla_{\parallel} \omega$$

$$\omega = \frac{1}{B} \nabla_{\perp}^2 \phi \quad , \quad B(r, \theta) = \frac{B_0}{1 + \frac{x}{R} \cos \theta}$$

$$c_s \nabla_{\parallel} n^i = \begin{cases} \frac{c_s}{L_{\parallel}} (n^{i+1}(r, \theta + \Delta\theta) - n^i(r, \theta)) & \text{if } n^i \geq n^{i+1} \\ \frac{c_s}{L_{\parallel}} (n^{i-1}(r, \theta - \Delta\theta) - n^i(r, \theta)) & \text{otherwise} \end{cases}$$

$$c_s \nabla_{\parallel} \omega^i = \begin{cases} \frac{c_s}{L_{\parallel}} (\omega^{i+1}(r, \theta + \Delta\theta) - \omega^i(r, \theta)) & \text{if } |\omega^i| \geq |\omega^{i+1}| \\ \frac{c_s}{L_{\parallel}} (\omega^{i-1}(r, \theta - \Delta\theta) - \omega^i(r, \theta)) & \text{otherwise} \end{cases}$$

$$\Delta\theta = \frac{2\pi}{qn_{\text{drift}}}$$

- Normalisation[4], space and time  $x \rightarrow \frac{x}{a}$ ,  $t \rightarrow \gamma t$
- Interchange growth rate:  $\gamma = \sqrt{\frac{2}{aR}} c_s$
- $c_s$  in normalized unites:  $c_s \rightarrow \sqrt{\frac{R}{2a}}$



# DIESEL

- Global model using full toroidal geometry on closed magnetic field lines
- Model, at present, based on a simple interchange model, see e.g. [1,2]
- $n_{\text{drift}}$  2-D drift planes each covering the full cross section of the torus
- In the above equations  $i \in [1; n_{\text{drift}}]$  and denotes the particular drift plane
- Parallel numerical code, based on spectral expansion of the solutions
- Scale linearly at least upto 100 CPU using 1024x2048 pr. drift plane
- Estimated maximum number of CPU is above 1.000!?! (to be tested)
- The drift planes are separated toroidally by  $L_{\square} = 2\pi R / n_{\text{drift}}$
- Parallel velocities are parameterized using  $c_s$  and  $V_A$
- $q$  enters in the two parallel terms:

